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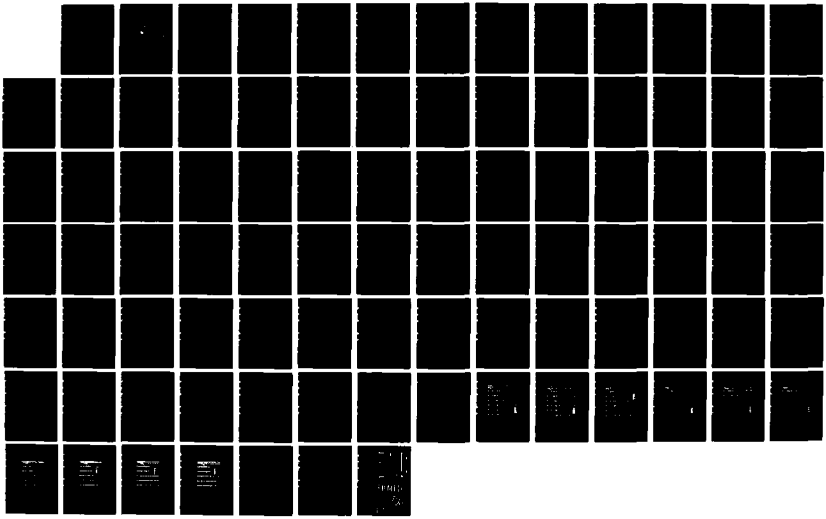

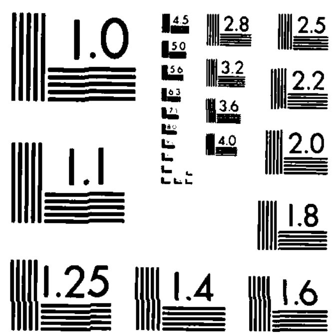

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THESIS

JAMMING EFFECTS ON DIGITAL COMMUNICATION RECEIVERS
(TIMING ERRORS AND FREQUENCY ERRORS)

By

Suk Ho Lee

December 1985

Thesis Advisor: D. Bukofzer

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derived and analyzed utilizing conventional binary modulation schemes. Results show that generally some but not a great deal of jammer effectiveness is lost due to random time arrival or random frequency errors associated with the jammer waveform.

Jamming Effects on Digital Communication Receivers (Timing Errors and Frequency Errors)

by

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Major, Republic of Korea Air Force
B.S., Korean Air Force Academy, 1977

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ABSTRACT

A refinement of the results obtained for optimum energy constrained jamming of digital receivers is obtained by modeling the jammer as a random process. In the modeling process, a random time arrival or random frequency errors are accounted for by including these effects in the representation of the jamming waveforms. Performance analyses are carried out in order to determine the effect of random time of arrival and random frequency errors on the part of the jammer, on the receiver probability of error.

The mathematical results derived are programmed, evaluated on the computer, and compared against ideal optimum energy constrained jamming strategies previously studied. Results for both coherent and incoherent receivers are derived and analyzed utilizing conventional binary modulation schemes. Results show that generally some but not a great deal of jammer effectiveness is lost due to random time of arrival or random frequency errors associated with the jammer waveform.

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I wish to express my sincere appreciation and indebtedness to Professor Daniel Bukofzer of the Electrical and Computer Engineering Department of the Naval Postgraduate School for the guidance, assistance and continuous encouragement which he provided during the pursuit of this project. This thesis is dedicated to my wife Yoonah and my children Jeewoo and Jeewon, for their support away from our home, during our life at Monterey in the United States.

I. INTRODUCTION

The structure and performance of digital receivers operating in additive White Gaussian Noise (WGN) is well known. While this noise assumption is often valid, in many cases, especially when there is intentional jamming in the channel, the WGN interference assumption breaks down. For this reason, there is interest in determining the performance of digital receivers designed to operate in a WGN environment, that must however operate in a different environment.

In this thesis, the performance of digital receivers has been analyzed under the assumption that the interference consists of additive WGN and some intentional jammer waveform, whose model represents a refinement over previous work in this area. It is to be expected that the presence of any jammer without such prior knowledge on the part of the receiver, will cause the performance of the receiver to be degraded.

For the case in which the jammer waveform is synchronized with the digital signal and has exact knowledge of the frequency of operation, results on receiver performance have already been obtained and analyzed [Ref 1,2]. This thesis endeavors to investigate the effect of a more realistic jammer model in which the jammer lacks synchronism with the digital signals as well as exact signal frequencies knowledge. This model is used in conjunction with both coherent and incoherent binary digital receivers designed to operate in a WGN environment.

In Chapter 2, the performance of coherent receivers is investigated under the assumption that the jammer waveform lacks synchronism with the digital signals or that it lacks exact knowledge of the frequencies of operation. A

mathematical model of the jammer waveform is introduced based on previous results on optimum energy constrained jammer waveforms used to degrade the performance of different binary modulation receivers. The analysis of the performance of the receivers is carried out and the effect of miss-synchronization (i.e, timing errors) and of frequency offsets (i.e, frequency errors) are discussed in Chapter 5.

In Chapter 3, the work carried out in Chapter 2 is repeated for incoherent receivers. Specifically, incoherent Amplitude Shift Keying (ASK) and incoherent Frequency Shift Keying (FSK) receivers are analyzed, performances are evaluated and the results are discussed in Chapter 5.

In Chapter 4, graphical results are presented corresponding to the numerical analyses that have been performed. In most cases, the graphs display receiver probability of error as a function of signal to noise ratio for given values of jammer to signal ratio. The effect of jammer miss-synchronization or of frequency offsets is then analyzed with the aid of the graphs.

Chapter 5 presents conclusions pertaining to what was learned from the results obtained and the numerical evaluations carried out.

II. COHERENT RECEIVERS

A. GENERAL

The correlator receiver structure depicted in Fig (5.1) which can be shown to be equivalent to the optimum single correlator receiver of Fig (5.2) is known to be optimum (in minimum probability of error sense) for discriminating between two signals $s_1(t)$ and $s_0(t)$, received in the presence of additive White Gaussian Noise (WGN), with $0 \leq t \leq T$. We define

$r(t)$ = received signal in the interval $0 \leq t \leq T$

$s_d(t) = s_1(t) - s_0(t)$

$N_o/2$ = Power spectral density level of the WGN.

P = probability that $s_0(t)$ is transmitted.

$$\gamma = \frac{N_o}{2} \cdot \ln\left(\frac{P}{1-P}\right) + \frac{1}{2}(E_1 - E_0)$$

where γ is the receiver threshold level used to decide on whether $s_1(t)$ or $s_0(t)$ was transmitted and E_i is the energy of the transmitted signal $s_i(t)$, that is

$$E_i = \int_0^T S_i(t)^2 \, dt \qquad i = 0,1$$

The probability of error P_e of this receiver is derived in many text books [See for example Ref 3]. The coherent digital communication receiver of Fig (5.2) can be analyzed in terms of the resulting P_e when $r(t)$ contains a jammer waveform $n_j(t)$ in addition to the noise and $s_0(t)$ or $s_1(t)$. Thus, under these conditions, the received signal appearing at the front end of the receiver is mathematically described by

$$r(t) = s_i(t) + n(t) + n_j(t) \qquad 0 \leq t \leq T \quad i = 0,1$$

where again $s_0(t)$ and $s_1(t)$ are the two signals used to transmit the binary information, $n(t)$ is a sample function of a White Gaussian Noise process having a power spectral density level of $N_o/2$ (Watts/Hz), and $n_j(t)$ is the jammer waveform present during the signaling interval [0,T].

The receiver of Figure (5.2) has been analyzed in so far as the effect of $n_j(t)$ on the receiver Probability of error (P_e) is concerned under the assumption that $n_j(t)$ can be modeled as deterministic waveform. [Ref 4]

It has been demonstrated that under such an assumption, the receiver P_e becomes

$$P_e = P \cdot \operatorname{erfc}\left[\frac{\gamma' + \frac{1}{2}||S_d||^2 - (n_j, S_d)}{\sqrt{\frac{No}{2}}\;||S_d||} \right]$$
$$+ (1-P)\ \operatorname{erf}\left[\frac{\gamma' - \frac{1}{2}||S_d||^2 - (n_j, S_d)}{\sqrt{\frac{No}{2}}\;||S_d||} \right] \qquad (2.1)$$

where

$$\gamma' = \frac{No}{2} \cdot \ln\frac{P}{1-P}$$
$$(n_j, S_d) = \int_0^T n_j(t) \cdot S_d(t)\, dt$$
$$\|s_d\|^2 = \int_0^T s_d(t)^2\, dt$$

With P=1/2 , we obtain

$$P_e = \frac{1}{2}\left[\int_{S_q\left[N_d - d\right]}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}}\, dx + \int_{-\infty}^{-S_q\left[N_d + d\right]} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}}\, dx \right] \qquad (2.2)$$

where

$$S_q = \left[\frac{2}{No\,||S_d||^2}\right] \qquad N_d = \frac{||S_d||^2}{2} \qquad d = (n_j, S_d)$$

B. JAMMER OPTIMIZATION

In the previous section, Equation (2.1) allows for the evaluation of performance of the receiver under the assumption that a jammer waveform is present. From the jammer standpoint, the optimum jammer waveform must be chosen in such a way that it maximizes the receiver Probability of error. By evaluating the first derivative and second derivative of P_e with respect to 'd' which is the cross correlation between the jammer waveform and the signal difference $s_d(t)$, the optimum jammer waveform can be chosen. Carrying out the mathematical operations, we have

$$\frac{\partial P_e}{\partial d} = \frac{S_q}{\sqrt{2\pi}} \cdot e^{-\frac{S_q^{\,2}\left[N_d^{\,2}+d^2\right]}{2}} \sinh(S_q^{\,2} \cdot N_d \cdot d)$$

$$\frac{\partial^2 P_e}{\partial\, d^2} = \frac{S_q^{\,3} N_d}{\sqrt{2\pi}}\, e^{-\frac{S_q^{\,3}\left[N_d+d\right]^2}{2}} > 0$$

From the above results, it can be seen that P_e is a monotonic function of 'd' and making 'd' as large as possible in magnitude results in the largest possible increase in P_e. In the limit, as $|d| \rightarrow \infty$ we have $P_e \rightarrow 1/2$. However, from the Cauchy-Schwarz inequality

$$||d|| = ||(n_j, s_d)|| \leq ||n_j||\ ||s_d|| \tag{2.3}$$

with equality if $n_j(t)$ is proportional to $s_d(t)$. Defining $||n_j|| = P_{nj}^{\,1/2}$, where P_{nj} is the jammer energy, the

condition $|d| \to \infty$ implies that $P_{nj} \to \infty$, when $||s_d|| < \infty$. Under the condition that the jammer power be constrained to P_{nj}, in order to get the maximum P_e, Equation (2.3) can be made into an equality by setting

$$n_j(t) = K\, s_d(t)$$

where K is a constant of proportionality. Since $||n_j||^2 = P_{nj}$, K must be set to the value $P_{nj}^{1/2}/||s_d||$. Thus $||d||$ is maximized by setting

$$n_j(t) = \frac{\sqrt{P_{nj}}}{||S_d||} \cdot S_d(t) \qquad (2.4)$$

and this results in P_e being maximized. Substituting Equation (2.4) into the probability of error expression of Equation (2.2) for P=1/2 yields

$$P_e = \frac{1}{2}\left[\int_{\sqrt{\frac{2}{No}}\left[\frac{||S_d||}{2} - \sqrt{P_{nj}}\right]}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \right.$$

$$\left. + \int_{\infty}^{-\sqrt{\frac{2}{No}}\left[\frac{||S_d||}{2} + \sqrt{P_{nj}}\right]} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \right] \qquad (2.5)$$

By defining

E/N_o = SNR ; signal to noise ratio.

P_{nj}/E = JSR ; jammer to signal ratio

and observing that

$$||S_d||^2 = \int_0^T [S_1(t) - S_0(t)]^2 dt = 2E(1-\rho)$$

where ρ is the cross correlation coefficient between two information signals, the probability of error expression of Equation (2.5) becomes

$$P_e = \frac{1}{2}\left[\int_{\sqrt{SNR}(\sqrt{1-\rho}\;-\sqrt{2JSR})}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + \int_{-\infty}^{-\sqrt{SNR}(\sqrt{1-\rho}\;+\sqrt{2JSR})} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx\right] \quad (2.6)$$

C. EFFECT OF DETERMINISTIC JAMMING WAVEFORM

In the previous section, the design of the jammer waveform and its degrading effects on the receiver performance has been analyzed. The effect of such jamming waveforms on the receiver performance has been found in terms of receiver probability of error. From Equation (2.6), it can be easily seen that the three independant variables which affect the performance of the receiver are SNR,JSR and cross correlation coefficient between the two signals used to transmit binary information, denote by ρ In a binary signaling set, it can be shown that $-1 \leq \rho \leq 1$. For Phase Shift Keying (PSK), ρ = -1 and for FSK, ρ = 0. Thus P_e for PSK, using Equation (2.6) is given by

$$P_e = \frac{1}{2}\left[\int_{\sqrt{SNR}(\sqrt{2}-\sqrt{2JSR})}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + \int_{-\infty}^{-\sqrt{SNR}(\sqrt{2}+\sqrt{2JSR})} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx\right] \quad (2.7)$$

and for FSK, using Equation (2.6), P_e is given by

$$P_e = \frac{1}{2}\left[\int_{\sqrt{SNR}(1-\sqrt{2JSR})}^{\infty} \frac{1}{2\pi} e^{-\frac{x^2}{2}} dx + \int_{-\infty}^{-\sqrt{SNR}(1+\sqrt{2JSR})} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx\right] \tag{2.8}$$

D. ANALYSIS OF THE EFFECT OF THE JAMMING WAVEFORM HAVING RANDOM TIME OF ARRIVAL

The results of the previous section demonstrate the effect of an energy constrained jammer on a binary communication receiver using a strictly deterministic jammer model. Such a model however must be refined because among other imperfections, the jammer signal may not be in synchronism with the digital transmission. In other words, there may be a difference in the time of arrival of the information signal and the jamming signal. The modeling of such an effect will be accomplished by letting

$$n_j(t) = K\ s_d(t-\tau) \qquad 0 \le \tau \le T \tag{2.9}$$

Observe that in this model, the optimum jamming waveshape has been maintained, however a time delay parameter τ has been introduced. In order to be able to analyze how the time delay τ influences the effectiveness of the jammer, and compare the resulting receiver P_e with that involving the idealized jammer model in which the jammer operates synchronously with the receiver, we will require that K be such that

$$||n_j||^2 = \int_0^T [K \cdot S_d(t-\tau)]^2\, dt = P_{nj} \tag{2.10}$$

Therefore, for this constraint to be satisfied, we must have

$$K = \sqrt{\frac{P_{nj}}{\int_{\emptyset}^{T} S_d^{\,2}(t-\tau)dt}} \tag{2.11}$$

and the jammer waveform is now modeled as

$$n_j(t) = K\, S_d(t-\tau) \tag{2.12}$$

with K given by Equation (2.11). In order to determine receiver performance using this new jammer model we may use Equation (2.1) in which now

$$(n_j, S_d) = K(S_{d,\tau}\ ,\ S_d) \tag{2.13}$$

where

$$(S_{d,\tau}\ ,\ S_d) = \int_{\emptyset}^{T} S_d(t-\tau)\cdot S_d(t)\ dt$$

while the remaining terms in Equation (2.1) are unchanged. Thus the P_e for the receiver operating in the presence of noise and a (non-synchronized) jammer mathematically modeled by Equation (2.12), is given by

$$P_e = P\cdot \mathrm{erfc}\left[\frac{\gamma' + \frac{1}{2}||S_d||^2 - K(S_{d,\tau}\ ,S_d)}{\sqrt{\frac{No}{2}}\ ||S_d||} \right]$$
$$+(1-P)\cdot \mathrm{erf}\left[\frac{\gamma' - \frac{1}{2}\ ||S_d||^2 - K(S_{d,\tau}\ ,S_d)}{\sqrt{\frac{No}{2}}\ ||S_d||} \right] \tag{2.14}$$

This expression can be evaluated for specific modulation schemes, such as Binary Phase Shift Keying (BPSK) and Binary Frequency Shift Keying (BFSK).

1. BPSK

For BPSK, the difference signal $s_d(t)$ defined on Page 12 becomes

$$s_d(t) = 2A \cos\omega_c t$$

where we assume that $\omega_c T = n\pi$ and 'n' is an integer. Thus

$$s_{d,\tau} = s_d(t-\tau) = 2A \cos\omega_c(t-\tau)$$

so that

$$\int_0^T S_d^2(t-\tau)dt = \int_0^T [\ 2A\cos\omega_c(t-\tau)\ ]^2\ dt = 2A^2T = 4E$$

where

$$A^2T/2 = E$$

The above assumption $\omega_c T = n\pi$ has been used in order to simplify the expression for the energy of $s_d(\ t-\tau\ )$. Thus from Equation (2.11) and Equation (2.12),

$$n_j(t) = \sqrt{\frac{P_{nj}}{4E}}\ S_d(t-\tau) = \frac{1}{2}\sqrt{JSR}\cdot S_d(t-\tau) \tag{2.15}$$

where we define

$$P_{nj}/E = JSR = \text{jammer to signal ratio}$$

In order to numerically evaluate Equation (2.14), we need to specify the inner product term, namely

$$\begin{aligned}(S_{d,\tau}, S_d) &= \int_0^T 2A\cos\omega_c(t-\tau)\cdot 2A\cos\omega_c t\ dt \\ &= 2A^2T\cdot\cos\omega_c\tau \end{aligned} \tag{2.16}$$

By substitution into Equation (2.14), receiver P_e for some given value of τ becomes

$$P_e = P \cdot \mathrm{erfc}\left[\frac{\ln \frac{P}{1-P} + \frac{1}{No} \|s_d\|^2 - \frac{2}{No} K \cdot 2A^2 T \cdot \cos\omega_c \tau}{\sqrt{\frac{2}{No}}\, \|s_d\|} \right] + (1-P) \cdot \mathrm{erf}\left[\frac{\ln \frac{P}{1-P} - \frac{1}{No} \|s_d\|^2 - \frac{2}{No} K \cdot 2A^2 T \cdot \cos\omega_c \tau}{\sqrt{\frac{2}{No}}\, \|s_d\|} \right] \tag{2.17}$$

where furthermore, since $\rho = -1$,

$$\frac{1}{No} \|s_d\|^2 = 2\ \mathrm{SNR}\ (1-\rho) = 4\ \mathrm{SNR}$$

$$\sqrt{\frac{2}{No}}\ \|s_d\|^2 = 2\sqrt{\mathrm{SNR}}$$

$$\frac{2}{No} K\ 2A^2 T = 4\ \mathrm{SNR}\sqrt{\mathrm{JSR}}$$

Thus, for the special case in which P=1/2,

$$P_e(\tau) = \frac{1}{2}\left[\mathrm{erfc}\left(\frac{4\mathrm{SNR} - 4\mathrm{SNR}\sqrt{\mathrm{JSR}} \cdot \cos\omega_c \tau}{2\sqrt{2\mathrm{SNR}}} \right) + \mathrm{erf}\left(\frac{-4\mathrm{SNR} - 4\mathrm{SNR}\sqrt{\mathrm{JSR}} \cdot \cos\omega_c \tau}{2\sqrt{2\mathrm{SNR}}} \right) \right] \tag{2.18}$$

Equation (2.18) can be expressed in terms of a normalized delay τ_N. Observe that

$$\cos\omega_c \tau = \cos(\omega_c T \cdot \frac{\tau}{T})$$

where

$$\tau_N = \frac{\tau}{T} \qquad (0 \le \tau_N \le T)$$

Thus Equation (2.18) can be reexpressed in terms of τ_N. The integer 'n' specifies the number of sine wave cycles within the observation interval. In most practical cases, n>10 , however results are not greatly affected by this parameter.

It is apparent that in most cases, τ_N cannot be determined a-priori, so it must be treated as a random quantity with some associated probability density function (p.d.f.). A logical, yet simple choice for such a p.d.f. is to assume τ_N uniformly distributed over [0,1], so that Equation (2.18) must be interpreted as receiver P_e conditioned on τ_N. The actual (unconditional) receiver P_e is obtained by integrating over the p.d.f. of τ_N, namely

$$
\begin{aligned}
P_e &= \int_0^1 P_e(\tau_N)\, d\tau_N \\
&= \frac{1}{2} \int_0^1 \Bigg[\operatorname{erfc}\left(\frac{4SNR - 4SNR\sqrt{JSR}\cdot\cos\omega_c\tau_N}{2\sqrt{2SNR}}\right) \\
&\quad + \operatorname{erf}\left(\frac{-4SNR - 4SNR\sqrt{JSR}\cdot\cos\omega_c\tau_N}{2\sqrt{2SNR}}\right) \Bigg] d\tau_N \qquad (2.19)
\end{aligned}
$$

2. <u>BFSK</u>

For BFSK, we have

$$s_d(t) = A\,(\cos\omega_1 t - \cos\omega_0 t)$$

where we assume that

$$\omega_s T = (\omega_1 + \omega_0)T = 2n\pi$$

$$\omega_d T = (\omega_1 - \omega_0)T = 2m\pi$$

and 'n','m' are integers with n>m. For this case,

$$s_{d,\tau} = s_d(t-\tau) = A\,(\cos\omega_1(t-\tau) - \cos\omega_0(t-\tau))$$

so that

$$\int_0^T s_d^{\,2}(t-\tau)\, dt = A^2 \left[\int_0^T [\cos\omega_1(t-\tau) - \cos\omega_0(t-\tau)]^2\, dt\right.$$

$$= A^2T = 2E \tag{2.20}$$

From Equations (2.11), (2.12) and (2.20) the jammer waveform becomes

$$n_j(t) = \sqrt{\frac{P_{nj}}{2E}} \cdot S_d(t-\tau) \tag{2.21}$$

Furthermore, the inner product term takes on the form

$$\begin{aligned}(S_{d,\tau}, S_d) &= \int_{0}^{T} A^2[\cos\omega_1(t-\tau) - \cos\omega_0(t-\tau)] \\ &\quad \cdot [\cos\omega_1 t - \cos\omega_0 t]\, dt \\ &= \frac{A^2T}{2} [\cos\omega_1\tau - \cos\omega_0\tau] \end{aligned} \tag{2.22}$$

so that substitution of these results in Equation (2.14), yields

$$\begin{aligned} P_e(\tau) = P\cdot \mathrm{erfc}&\left[\frac{\ln\frac{P}{1-P} + 2SNR - \sqrt{2JSR}\cdot SNR\cdot(\cos\omega_1\tau - \cos\omega_0\tau)}{2\sqrt{SNR}} \right] \\ +(1-P)\cdot \mathrm{erf}&\left[\frac{\ln\frac{P}{1-P} - 2SNR - \sqrt{2JSR}\cdot SNR\cdot(\cos\omega_1\tau - \cos\omega_0\tau)}{2\sqrt{SNR}} \right] \end{aligned} \tag{2.23}$$

Utilizing the previously defined normalized delay τ_N, Equation (2.23) becomes

$$\begin{aligned} P_e(\tau_N) = P\cdot \mathrm{erfc}&\left[\frac{\ln\frac{P}{1-P} + 2SNR - \sqrt{2JSR}\cdot SNR\cdot(2\cos n\pi\tau_N \cdot \cos m\pi\tau_N)}{2\sqrt{SNR}} \right] \\ +(1-P)\cdot \mathrm{erf}&\left[\frac{\ln\frac{P}{1-P} - 2SNR - \sqrt{2JSR}\cdot SNR(2\cos n\pi\tau_N \cdot \cos m\pi\tau_N)}{2\sqrt{SNR}} \right] \end{aligned} \tag{2.24}$$

where

$0 \le \tau_N \le 1$

Assuming once again that τ_N must be treated as a random quantity with uniform p.d.f. over [0,1], for the special case of P=1/2, we obtain

$$P_e(\tau_N) = \int_0^1 P_e(\tau_N)\, d\tau_N$$

$$= \frac{1}{2} \int_0^1 \Big[\operatorname{erfc}\cdot\Big(\frac{2SNR - 2SNR\sqrt{2JSR}\cdot \cos n\pi\tau_N \cdot \cos m\pi\tau_N}{2\sqrt{SNR}} \Big)$$

$$+ \operatorname{erf}\cdot\Big(\frac{-2SNR - 2SNR\sqrt{2JSR}\cdot \cos n\pi\tau_N \cdot \cos m\pi\tau_N}{2SNR} \Big) \Big] d\tau_N \tag{2.25}$$

E. ANALYSIS OF THE EFFECT OF THE JAMMING WAVEFORM HAVING RANDOM FREQUENCY ERRORS

In actual situations, it is oftentimes difficult for the communication jammer to know the exact transmission frequency or frequencies of the adversary's communication system. This lack of knowledge may be due to the use of imprecise frequency estimation methods, or due to frequent frequency changes made by the communication system's users. Thus in this section the effect of lack of precise transmission frequency knowledge on the part of jammer is studied and analyzed insofar as coherent binary digital receivers is concerned. Specifically, BPSK and BFSK modulation only, will be considered.

1. BPSK

The lack of precise transmission frequency knowledge on the part of the jammer is modeled by introducing a frequency difference $\Delta\omega$ in the known optimum jammer model for BPSK. That is,

$$n_j(t) = K \cos(\omega_c + \Delta\omega)t \tag{2.26}$$

where $\Delta\omega$ is, in a sense, the jammer frequency error. Since the jammer power is constrained to be P_{nj}, we must have

$$\|n_j\|^2_{\Delta\omega} = P_{nj} = \int_0^T [K\cos(\omega_c+\Delta\omega)t]^2 dt$$
$$= \frac{K^2 T}{2}\left[1 + \frac{\sin 2(\omega_c+\Delta\omega)T}{2(\omega_c+\Delta\omega)T}\right] \tag{2.27}$$

Thus we have from Equation (2.11),

$$K = \left[\frac{2\ P_{nj}}{T\left[1 + \frac{\sin 2(\omega_c+\Delta\omega)T}{2(\omega_c+\Delta\omega)T}\right]}\right]^{\frac{1}{2}} \tag{2.28}$$

In order to determine the effect of this jammer on the coherent receiver, all that is needed is the determination of the inner product $(n_j,s_d)_{\Delta\omega}$. Thus

$$(n_j,S_d)_{\Delta\omega} = \int_0^T K\cos(\omega_c+\Delta\omega)t\cdot 2A\cos\omega_c t\cdot dt$$
$$= KAT\left[\frac{\sin\Delta\omega T}{\Delta\omega T} + \frac{\sin(2\omega_c+\Delta\omega)T}{(2\omega_c+\Delta\omega)T}\right] \tag{2.29}$$

Defining now a normalized frequency error, $\Delta\omega_N$, where

$$\Delta\omega_N = \frac{\Delta\omega}{\omega_c}$$

the constant K and $(n_j,s_d)_{\Delta\omega}$ can be rewritten as

$$K = \left[\frac{2\ P_{nj}}{T\left[1 + \frac{\sin 2\omega_c T(1+\omega_N)}{2\omega_c T(1+\omega_N)}\right]}\right]^{\frac{1}{2}}$$

and

$$
\begin{aligned}
(n_j, S_d)_{\Delta\omega} &= KAT \cdot \left[\frac{\sin\omega_c T \cdot \Delta\omega_N}{\omega_c T \cdot \Delta\omega_N} + \frac{\sin\omega_c T(2+\Delta\omega_N)}{\omega_c T(2+\Delta\omega_N)} \right] \\
&= A\sqrt{T} \left[\frac{2\,P_{nj}}{T\left[1 + \dfrac{\sin 2\omega_c T(1+\omega_N)}{2\omega_c T(1+\omega_N)}\right]} \right]^{\frac{1}{2}} \\
&\quad \cdot \left[\frac{\sin\omega_c T \cdot \Delta\omega_N}{\omega_c T \cdot \Delta\omega_N} + \frac{\sin\omega_c T(2+\Delta\omega_N)}{\omega_c T(2+\Delta\omega_N)} \right]
\end{aligned}
\tag{2.30}
$$

Substituting Equation (2.30) in Equation (2.1), we obtain

$$
\begin{aligned}
P_e(\Delta\omega) &= P \cdot \mathrm{erfc} \cdot \left[\frac{\ln\frac{P}{1-P} + 4SNR - 4SNR\sqrt{JSR}\cdot[B]\cdot[C]}{2\sqrt{2SNR}} \right] \\
&\quad + (1-P)\cdot \mathrm{erf} \cdot \left[\frac{\ln\frac{P}{1-P} - 4SNR - 4SNR\sqrt{JSR}\cdot[B]\cdot[C]}{2\sqrt{2SNR}} \right]
\end{aligned}
\tag{2.31}
$$

where

$$
B = \left[\frac{1}{1 + \dfrac{\sin\omega_c T(1+\Delta\omega_N)}{\omega_c T(1+\Delta\omega_N)}} \right]^{\frac{1}{2}}
$$

$$
C = \left[\frac{\sin\omega_c T \cdot \Delta\omega_N}{\omega_c T \cdot \Delta\omega_N} + \frac{\sin\omega_c T(2+\Delta\omega_N)}{\omega_c T(2+\Delta\omega_N)} \right]
$$

Equation (2.31) yields the receiver P_e when $\Delta\omega_N$ has some known value. Unfortunately, in many cases $\Delta\omega_N$ will not be known exactly and must therefore be modeled as a random variable. Thus, in order to compute the average value of P_e, it is assumed that the probability density of $\Delta\omega_N$ is uniform on $(\Delta\omega_L, \Delta\omega_H)$, where $\Delta\omega_L$ is the lower limit and $\Delta\omega_H$ is the upper limit of the normalized frequency error. With this assumption, the average P_e can be calculated as

$$P_e = \frac{1}{\Delta\omega_H - \Delta\omega_L} \int_{\Delta\omega_L}^{\Delta\omega_H} P_e(\Delta\omega_N) \cdot d\Delta\omega_N \tag{2.32}$$

or equivalently

$$P_e = \frac{1}{\Delta\omega_H - \Delta\omega_L} \int_{\Delta\omega_L}^{\Delta\omega_H} \{ P \cdot \mathrm{erfc} \cdot \left[\frac{\ln\frac{P}{1-P} + 4SNR - 4SNR\sqrt{JSR} \cdot [B] \cdot [C]}{2\sqrt{2SNR}} \right]$$

$$+ (1-P) \cdot \mathrm{erf} \cdot \left[\frac{\ln\frac{P}{1-P} - 4SNR - 4SNR\sqrt{JSR} \cdot [B] \cdot [C]}{2\sqrt{2SNR}} \right] \} d\Delta\omega_N$$

$$\tag{2.33}$$

where B and C have been previously specified and are functions of $\Delta\omega_N$. Results on the evaluation of Equation (2.33) will be presented in Chapter 4, under the assumption that P=1/2.

2. BFSK

The lack of precise knowledge of the transmission frequencies on the part of the jammer, is modeled by introducing frequency differences, $\Delta\omega_1$ and $\Delta\omega_0$ in the known optimum jammer model for BFSK. That is

$$n_j(t) = K[\cos(\omega_1 + \Delta\omega_1)t - \cos(\omega_0 + \Delta\omega_0)t]$$

where in a sense, $\Delta\omega_1$ and $\Delta\omega_0$ are the jammer frequency errors. Since the jammer power is constrained to be P_{nj}, we have

$$\|n_j\|^2 = P_{nj} = \int_0^T \left[K[\cos(\omega_1+\Delta\omega_1)t - \cos(\omega_0+\Delta\omega_0)t]^2 dt \right.$$

$$= \frac{K^2 T}{2} \left[2 + \frac{\sin 2(\omega_1+\Delta\omega_1)T}{2(\omega_1+\Delta\omega_1)T} + \frac{\sin 2(\omega_0+\Delta\omega_0)T}{2(\omega_0+\Delta\omega_0)T} \right.$$

$$\left. -2\frac{\sin(\omega_1+\Delta\omega_1+\omega_0+\Delta\omega_0)T}{(\omega_1+\Delta\omega_1+\omega_0+\Delta\omega_0)T} - 2\frac{\sin(\omega_1+\Delta\omega_1-\omega_0-\Delta\omega_0)T}{(\omega_1+\Delta\omega_1-\omega_0-\Delta\omega_0)T} \right]$$

Thus

$$K=\left[\frac{2P_{nj}}{T(2+\mathrm{sinc}2(\omega_1+\Delta\omega_1)T+\mathrm{sinc}2(\omega_0+\Delta\omega_0)T-\mathrm{sinc}(\omega_s+\Delta\omega_s)T-2\mathrm{sinc}(\omega_d-\omega_d))}\right]^{\frac{1}{2}}$$

$$=\sqrt{\frac{2P_{nj}}{T}}\cdot[D]$$

where

$$\mathrm{sinc}\ x=\frac{\sin x}{x}$$

$$\omega_s=\omega_1+\omega_0$$

$$\omega_d=\omega_1+\omega_0$$

and

$$D=\left[\frac{1}{T(2+\mathrm{sinc}2(\omega_1+\Delta\omega_1)T+\mathrm{sinc}2(\omega_0+\Delta\omega_0)T-\mathrm{sinc}(\omega_s+\Delta\omega_s)T-2\mathrm{sinc}(\omega_d-\omega_d))}\right]^{\frac{1}{2}}$$

In order to determine the effect of this jammer on the coherent receiver, all that is needed is the determination of the inner product (n_j,s_d). Thus

$$(n_j,S_d)=\int_0^T K[\cos(\omega_1+\Delta\omega_1)t-\cos(\omega_0+\Delta\omega_0)t]\cdot A[\cos\omega_1 t-\cos\omega_0 t]dt$$

$$=\frac{KAT}{2}\cdot\left[\frac{\sin\Delta\omega_1 T}{\Delta\omega_1 T}+\frac{\sin(2\omega_1+\Delta\omega_1)T}{(2\omega_1+\Delta\omega_1)T}-\frac{\sin(\omega_d-\Delta\omega_0)T}{(\omega_d-\Delta\omega_0)T}\right.$$
$$-\frac{\sin(\omega_s+\Delta\omega_0)T}{(\omega_s+\Delta\omega_0)T}-\frac{\sin(\omega_d+\Delta\omega_1)T}{(\omega_d+\Delta\omega_1)T}-\frac{\sin(\omega_s+\Delta\omega_1)T}{(\omega_s+\Delta\omega_1)T}$$
$$\left.+\frac{\sin(2\omega_0+\Delta\omega_0)T}{(2\omega_0+\Delta\omega_0)T}+\frac{\sin\Delta\omega_0 T}{\Delta\omega_0 T}\right]$$

We define now two normalized frequency estimation errors, namely

$$\Delta\omega_N=\frac{\Delta\omega_i}{\omega_i}\qquad i=0,1$$

so that

$$\Delta\omega_1 T=\omega_1 T\ \Delta\omega_{N,1}$$

$$\Delta\omega_0 T=\omega_0 T\ \Delta\omega_{N,0}$$

and

$$(n_j, S_d) = \frac{EAT}{2} \cdot [F] \tag{2.34}$$

where

$$\begin{aligned} F = {} & \mathrm{sinc}(\omega_1 T \cdot \Delta\omega_{N,1}) + \mathrm{sinc}(\omega_1 T \cdot (2+\Delta\omega_{N,1})) \\ & - \mathrm{sinc}(\omega_d T - \omega_0 T \cdot \Delta\omega_{N,0}) - \mathrm{sinc}(\omega_s T - \omega_0 T \cdot \Delta\omega_{N,0}) \\ & - \mathrm{sinc}(\omega_d T + \omega_1 T \cdot \Delta\omega_{N,1}) - \mathrm{sinc}(\omega_s T + \omega_1 T \cdot \Delta\omega_{N,1}) \\ & + \mathrm{sinc}(\omega_0 T \cdot \Delta\omega_{N,0}) + \mathrm{sinc}(\omega_0 T(2+\Delta\omega_{N,0})) \end{aligned}$$

Using again Equation (2.1), $Pe(\Delta\omega_{N,1}, \Delta\omega_{N,0})$ for known $\Delta\omega_{N,1}$ and $\Delta\omega_{N,0}$ values become

$$\begin{aligned} P_e(\Delta\omega_{N,1}, \Delta\omega_{N,0}) = {} & P \cdot \mathrm{erfc}\left[\frac{\ln\frac{P}{1-P} + 2SNR - 2SNR\sqrt{JSR} \cdot [D] \cdot [F]}{2\sqrt{SNR}}\right] \\ & + (1-P) \cdot \mathrm{erf}\left[\frac{\ln\frac{P}{1-P} - 2SNR - 2SNR\sqrt{JSR} \cdot [D] \cdot [F]}{2\sqrt{SNR}}\right] \end{aligned} \tag{2.35}$$

If we now treat $\Delta\omega_{N,1}$ and $\Delta\omega_{N,0}$ as random variables uniformly distributed over appropriate ranges, and furthermore, assume that the normalized frequency errors are statistically independent, the probability density function of $\Delta\omega_{N,1}$ and $\Delta\omega_{N,0}$ becomes

$$p(\Delta\omega_{N,1}, \Delta\omega_{N,0}) = \frac{1}{(\Delta\omega_{N,1,H} - \Delta\omega_{N,1,L})(\Delta\omega_{N,0,H} - \Delta\omega_{N,0,L})}$$

where $\Delta\omega_{N,1}$ is in the region $(\Delta\omega_{N,1,L}, \Delta\omega_{N,1,H})$, and $\Delta\omega_{N,0}$ is in the region $(\Delta\omega_{N,0,L}, \Delta\omega_{N,0,H})$. The average P_e is finally obtained from

$$P_e = \int_{\Delta\omega_{N,1,L}}^{\Delta\omega_{N,1,H}} \int_{\Delta\omega_{N,0,L}}^{\Delta\omega_{N,0,H}} P_e(\Delta\omega_{N,1}, \Delta\omega_{N,0}) \cdot p(\Delta\omega_{N,1}, \Delta\omega_{N,0}) \cdot d\Delta\omega_{N,1} d\Delta\omega_{N,0}$$

or equivalently

$$P_e = \frac{1}{(\Delta\omega_{N,1,H} - \Delta\omega_{N,1,L})(\Delta\omega_{N,\emptyset,H} - \Delta\omega_{N,\emptyset,L})} \int_{\Delta\omega_{N,1,L}}^{\Delta\omega_{N,1,H}} \int_{\Delta\omega_{N,\emptyset,L}}^{\Delta\omega_{N,\emptyset,H}}$$

$$\left[P \cdot \mathrm{erfc}\left[\frac{\ln\frac{P}{1-P} + 2SNR - 2SNR\sqrt{JSR}\cdot[D]\cdot[F]}{2\sqrt{SNR}}\right] + (1-P)\cdot \mathrm{erf}\left[\frac{\ln\frac{P}{1-P} - 2SNR - 2SNR\sqrt{JSR}\cdot[D]\cdot[F]}{2\sqrt{SNR}}\right]\right] d\Delta\omega_{N,1}\, d\Delta\omega_{N,\emptyset} \quad (2.36)$$

The results of the evaluation of this Probability of error expression will be shown in Chapter 4.

III. INCOHERENT RECEIVERS

A. GENERAL

In incoherent systems, the phase of the carrier signals is not available at the receiver so that the phase must be treated as a random variable which is typically assumed to be uniformly distributed over $[0, 2\pi]$. As such, we may expect the performance of an incoherent receiver to be degraded in comparison to the performance of the corresponding coherent receiver. However, because of their simplicity, incoherent systems are widely used in many applications.

The analysis to be carried out on the performance of an incoherent receiver in the presence of WGN and jamming is based on previous work in which it was assumed that a deterministic jammer model was adequate, and that the optimum energy constrained jamming waveform for the corresponding coherent receiver can act as a good jammer for the incoherent receiver also. Thus, such near optimum jammer signals are studied and evaluated in terms of their effect on the performance of incoherent receivers, under more realistic conditions now in which the jammer is miss-synchronized, or lacks exact knowledge of the signal's operating frequencies.

B. EFFECTS OF OPTIMUM JAMMING WAVEFORMS

1. ASK

Analysis of the incoherent receiver starts from the assumption that r(t), the signal appearing at the front end of the receiver can be mathematically modeled by either

$$r(t) = A \sin(\omega_c t + \Theta) + n(t) + n_j(t) \quad 0 \le t \le T$$

or

$$r(t) = n(t) + n_j(t) \qquad 0 \le t \le T$$

where Θ is a random variable uniformly distributed over $[0,2\pi]$, $n(t)$ is a sample function of a White Gaussian Noise process having power spectral density level $N_o/2$ (Watts/Hz), and $n_j(t)$ is the jamming waveform present during the interval $[0,T]$.

In the absence of $n_j(t)$, the optimum receiver for the binary ASK problem being presented is well-known. Its derivation is well documented in the statistical detection theory literature [Ref 3]. The receiver structure is shown in Figure (5.3). An equivalent form of this structure is shown in Figure (5.4).

In this section, the effect of $n_j(t)$ on this receiver is analyzed by evaluating the resulting P_e, under the assumption that $n_j(t)$ is a deterministic waveform, however unknown to the receiver itself. This analysis has been carried out in [Ref 2], but is repeated here for completeness and the general result is applied to the specific problem analyzed in this thesis. Receiver performance evaluation requires determination of the statistics of either G^2 or G, where G^2 is the output of the quadrature detector and is given by

$$G^2 = X^2 + Y^2$$

where

$$X = \int_0^T r(t) \cdot \sin\omega_c t \cdot dt = (r,S)$$

and

$$Y = \int_0^T r(t) \cdot \cos\omega_c t \cdot dt = (r,C)$$

Provided that the random variable Θ is fixed to some value θ , X and Y are conditional Gaussian random variables with

$$E\{X|H_1,\theta\} = (AS_\theta,S) + (n_j,S) = m_{X|\theta}$$
$$E\{Y|H_1,\theta\} = (AS_\theta,S) + (n_j,S) = m_{Y|\theta} \tag{3.1}$$

where S_θ is used in place of $\sin(\omega_c t + \theta)$. Also, it can be shown that

$$Var\{X|H_1,\theta\} = E\{(n,S)^2\} = \frac{NoT}{4}\left[1- \frac{\sin 2\omega_c T}{2\omega_c T}\right] = Var\{X|H_0\}$$

and similarly

$$Var\{Y|H_1,\theta\} = \frac{NoT}{4}\left[1- \frac{\sin 2\omega_c T}{2\omega_c T}\right] = Var\{Y|H_0\}$$

For convenience, we assume $\omega_c T = n\pi$, where 'n' is an integer. Thus the 'sinc' terms above vanish resulting in the simplification

$$Var\{X|H_1,\theta\} = Var\{Y|H_1,\theta\} = \frac{NoT}{4} = \sigma^2$$

If this assumption is not made, an additional term remains. However, if $(\pi/\omega_c) << T$, the additional 'sinc' term is small and can be neglected. Furthermore, the covariance

$$E\{ [X - E\{X|H_i,\theta\}]\cdot[Y - E\{Y|H_i,\theta\}] \mid H_i,\theta\}$$
$$= \frac{NoT}{4}\left[\frac{1- \sin 2\omega_c T}{2\omega_c T} \right] = 0 \qquad i=0,1$$

provided the assumption on ω_c holds. This implies that for any given value of θ, both X and Y are uncorrelated Gaussian random variables and therefore statistically independent. The density function of G^2 conditioned on the phase θ is non-central Chi-Squared distributed, and is given by

$$p_{G^2|\theta}(g|H_1,\theta) = \frac{1}{2\sigma^2}\cdot e^{-\frac{g+M}{2\sigma^2}}\cdot I_o\left(\frac{\sqrt{gM}}{\sigma^2}\right) \qquad g\geq 0$$

and zero otherwise, where σ^2 is defined above and

$$M = E^2(X|H_1,\theta) + E^2(Y|H_1,\theta)$$
$$= [\ (AS_\theta,S) + (n_j,S)\ ]^2 + [\ (AS_\theta,C) + (n_j,C)\ ]^2$$

Due to our assumption $\omega_c t=n\pi$, we obtain

$$M = (\frac{AT}{2})^2 + AT[(n_j,S)\cos\theta + (n_j,C)\sin\theta] + (n_j,S)^2 + (n_j,C)^2 \qquad (3.2)$$

If r(t) consists of noise and jammer only, then X and Y are also independent Gaussian random variables with

$$E\{X|H_\emptyset\} = (n_j,S) = m_X$$
$$E\{Y|H_\emptyset\} = (n_j,C) = m_Y$$

so that the density function of G^2 assuming no signal is sent is

$$p_{G^2}(g|H_\emptyset) = \frac{1}{2\sigma^2}\, e^{-\frac{g+M'}{2\sigma^2}} \cdot I_o(\frac{\sqrt{gM'}}{\sigma^2}) \qquad g \geq \emptyset$$

and zero otherwise, where

$$M' = (n_j,S)^2 + (n_j,C)^2 \qquad (3.3)$$

If we now let

$P(H_1)$ = Pr {sinusoid transmitted}

$P(H_\emptyset)$ = Pr {no signal transmitted}

then

$$P_e = P(H_1)\int_0^{\eta^2} p_{G^2}(g|H_1)dg + P(H_0)\int_{\eta^2}^{\infty} p_{G^2}(g|H_0)dg \qquad (3.4)$$

where

$$p_{G^2}(g|H_i) = \int_{-\infty}^{\infty} p_{G^2}(g|H_i,\theta)\cdot p_\theta(\theta)\, d\theta \qquad i=0,1$$

The second integral in Equation (3.4) can be expressed as follows

$$\int_{\eta^2}^{\infty} \frac{1}{2\sigma^2}\cdot e^{-\frac{g+M'}{2\sigma^2}} \cdot I_o\left(\frac{\sqrt{gM'}}{\sigma^2}\right)\cdot dg$$

$$= \int_{\frac{\eta}{\sigma}}^{\infty} V\cdot e^{-\frac{V^2+M'}{2}} I_o\left(\frac{\sqrt{M'}}{\sigma} V\right)\, dV = Q\left(\frac{\sqrt{M'}}{\sigma}, \frac{\eta}{\sigma}\right)$$

where

$$Q(\alpha,\beta) = \int_{\beta}^{\infty} V\cdot e^{-\frac{V^2+\alpha^2}{2}} I_o(\alpha V)\, dV$$

is a well-known tabulated function called the Marcum Q function. The first integral in Equation (3.4) can be expressed as follows

$$\int_0^{\eta^2}\left[\int_{-\infty}^{\infty} p_{G^2}(g|H_1,\theta)\; p_\theta(\theta)\cdot d\theta\right]\cdot dg$$

$$= \int_{-\infty}^{\infty} p_\theta(\theta)\left[\int_0^{\eta^2} p_{G^2}(g|H_1,\theta)\right] d\theta \qquad (3.5)$$

where the Equation (3.5) has been obtained by changing the order of integration. The inner integral of Equation (3.5) can be expressed in terms of the Marcum Q function as

$$\int_{0}^{\eta^2} \frac{1}{2\sigma^2} \cdot e^{-\frac{g+M}{2\sigma^2}} \cdot I_o\left(\frac{\sqrt{gM}}{\sigma^2}\right) \cdot dg$$

$$= \int_{0}^{\frac{\eta}{\sigma}} V \cdot e^{-\frac{V^2+M}{2}} \cdot I_o\left(\frac{\sqrt{M}}{\sigma} \cdot V\right) \, dV = 1 - Q\left(\frac{\sqrt{M}}{\sigma}, \frac{\eta}{\sigma}\right)$$

Therefore the receiver probability of error P_e can be written as

$$P_e = P(H_0) \cdot Q\left(\frac{\sqrt{M'}}{\sigma}, \frac{\eta}{\sigma}\right) + P(H_1)\left[1 - \frac{1}{2} \cdot \int_{0}^{2\pi} Q\left(\frac{\sqrt{M}}{\sigma}, \frac{\eta}{\sigma}\right) d\theta\right] \quad (3.6)$$

where the dependence on θ is imbedded in the terms M and the M', the threshold η which the receiver sets assuming that no jammer is present is obtained as the solution to the equation

$$e^{-\frac{A^2T}{No}} \cdot I_o\left(\frac{2A\eta}{No}\right) = \lambda_0 \qquad \lambda_0 = \frac{P(H_0)}{P(H_1)}$$

which can be equivalently put in the form

$$e^{-\frac{A^2T}{No}} \cdot I_o\left(\sqrt{\frac{A^2T}{No}} \cdot \frac{\eta}{\sigma} \right) = \lambda_0$$

The I(•) function used here and also previously used in conjunction with the development of P_e is the modified Bessel function of the first kind. If the definition of average signal energy previously introduced is used, we have

$E = (A^2T)/4$

which is reduced in half in comparison to the signal transmission case, due to the fact that the information bearing signal(s) do not have equal energy.

In order to afford comparisons with the coherent receiver case, we implicitly boost the value of the signal amplitude A, to obtain

$$E = (A^2T)/2$$

in order to have agreement with previous cases insofar as signal energy is concerned. Thus the threshold determination equation now becomes

$$e^{-SNR} \cdot I_0\left(\sqrt{2SNR} \cdot \frac{\eta}{\sigma} \right) = \lambda_0$$

If we assume $P(H_1) = P(H_0) = 1/2$, then

$$P_e = \frac{1}{2}\left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\frac{\sqrt{M}}{\sigma}, \frac{\eta}{\sigma} \right) d\theta + Q\left(\frac{\sqrt{M'}}{\sigma}, \frac{\eta}{\sigma} \right) \right] \qquad (3.7)$$

and the threshold setting equation becomes

$$e^{-SNR} \cdot I_0\left(\sqrt{2SNR} \cdot \frac{\eta}{\sigma} \right) = 1$$

If we are to find the optimum jammer waveform so as to maximize P_e, an attempt must be made to solve

$$\frac{\partial P_e}{\partial M} = 0 \qquad \text{and} \qquad \frac{\partial P_e}{\partial M'} = 0$$

Unfortunately the resultant equations are mathematically involved and do not appear readily solvable for $n_j(t)$. It seems however that a good jammer waveform can be postulated based on the results obtained for coherent ASK. It was found for such a case that the optimum $n_j(t)$ under the constraint that the energy of $n_j(t)$ be limited to

some value P_{nj} is a tone at the carrier frequency. Thus the following jammer waveform can be used as a potential near optimum jammer, namely

$$n_j(t) = \sqrt{P_{nj}\frac{2}{T}} \cdot \sin\omega_c t \qquad 0 \le t \le T \qquad (3.8)$$

Observe that with this choice, $||n_j(t)|| = P_{nj}$. The probability of error P_e can now be determined using the threshold setting equation and the previously derived expressions for M and M'. The effect of the near optimum jammer waveform on the receiver (i.e, incoherent receiver performance) can be analyzed by evaluating P_e as a function of JSR using Equation (3.8).

It can be shown that when $P(H_0) = P(H_1) = 1/2$ the probability of receiver error given by Equation (3.6) becomes

$$P_e = \frac{1}{2}\left[1+Q(\sqrt{2\cdot SNR\cdot JSR},\frac{\eta}{\sigma}) - \frac{1}{2\pi}\int_0^{2\pi} Q(\sqrt{2\cdot SNR(1+2\sqrt{JSR}\cdot\cos\theta+JSR)},\frac{\eta}{\sigma})\cdot d\theta \right] \qquad (3.9)$$

where

$$JSR = P_{nj}/E$$

In section C and D of this chapter, P_e will be evaluated once again under a more realistic jammer model that includes miss-synchronization and frequency offsets.

2. BFSK

For binary incoherent FSK with a jammer present, the received signals under the two hypotheses are either

$$H_1 \; ; \; r(t) = A\cdot\sin(\omega_1 t + \Theta) + n(t) + n_j(t) \qquad 0 \le t \le T$$

or

$$H_0 \; ; \; r(t) = A \cdot \sin(\omega_0 t + \phi) + n(t) + n_j(t) \qquad 0 \le t \le T$$

By separating the frequencies ω_1 and ω_0 sufficiently, we can form signals that are orthogonal, have equal energy, and have the same advantage of ease of generation. Here, θ and π are assumed to be independent random variables uniformly distributed over $[0,2\pi]$.

The receiver function is to compare the envelopes at the output of each channel once every T seconds and decide in favor of the larger of the two envelopes (Figure 5.5). It is assumed that the probability of sending either one of the two signals is 0.5. For the purpose of analysis, assume first that a 'mark' signal has been transmitted, that is, the hypothesis H_1 is true. An error is committed if q_0 exceeds q_1. An error is also committed if q_1 is larger than q_0 when a 'space' signal has been transmitted, that is, when the hypothesis H_0 is true.[Ref 3]

Let P_{e1} denote the probability of the first type of error described above, which is expressed as $Pr(q_0 > q_1 \mid H_1)$. Under the assumption that a 'mark' signal has been sent, the output q_1 of one of the envelope detectors is given by

$$q_1^2 = X_1^2 + Y_1^2$$

where

$$X_1 = \int_0^T r(t) \sin\omega_1 t \, dt \qquad\qquad Y_1 = \int_0^T r(t) \cos\omega_1 t \, dt$$

Observe that X_1 and Y_1 conditioned on the phase and either of the two hypotheses are Gaussian random variables with

$$E(X_1 | H_1, \theta) = \int_0^T A \cdot \sin(\omega_1 t + \theta) \cdot \sin\omega_1 t \cdot dt + \int_0^T n_j(t) \cdot \sin\omega_1 t \, dt$$

$$\equiv (AS_{1,\theta}, S_1) + (n_j, S_1)$$

and

$$E\{Y_1|H_1,\theta\} = \int_0^T A\cdot\sin(\omega_1 t+\theta)\cdot\cos\omega_1 t\cdot dt + \int_0^T n_j(t)\cdot\cos\omega_1 t\, dt$$

$$\equiv (AS_{1,\theta},C_1) + (n_j,C_1)$$

where $AS_{1,\theta}$ represents the function $A\sin(\omega_1 t + \theta)$. S_1 represents the function $\sin\omega_1 t$ and C_1 represents the function $\cos\omega_1 t$. Likewise, assuming again that $\omega_1 T = n\pi$, where 'n' is an integer, and that n(t) is a zero mean white Gaussian noise process with P.S.D. level $N_o/2$ Watts/Hz, we obtain

$$\mathrm{Var}\{X_1|H_1,\theta\} = \mathrm{Var}\{Y_1|H_1,\theta\} = \frac{NoT}{4}$$

Furthermore, it can be shown that

$$E\{\ [X_1 - E\{X_1|H_1,\theta\}]\cdot[Y_1 - E\{Y_1|H_1,\theta\}]\ |\ H_1,\theta\ \} = 0$$

so that the conditionally Gaussian random variables X_1 and Y_1 are uncorrelated and therefore independent. The sum involving random variables X_1 and Y_1, and producing $q_1{}^2$, will result in a non-central Chi-Squared distribution so that

$$p_{Q_1^2|H_1,\theta}(q_1|H_1,\theta) = \frac{1}{2\sigma^2}\cdot e^{-\frac{q_1+m_{11}}{2\sigma^2}}\cdot I_o\left(\frac{\sqrt{q_1\cdot m_{11}}}{\sigma^2}\right) \qquad q_1\geq 0$$

where

$$m_{11} = E^2\{X_1|H_1,\theta\} + E^2\{Y_1|H_1,\theta\}$$

$$= [(AS_{1,\theta},S_1) + (n_j,S_1)]^2 + [(AS_{1,\theta},C_1) + (n_j,C_1)]^2$$

and $\sigma^2 = N_o T/4$. Using standard random variable transformation techniques, it can be shown that

$$p_{Q_1|H_1,\theta}(q_1|H_1,\theta) = \frac{q_1}{\sigma^2}\cdot e^{-\frac{q_1^2+m_{11}}{2\sigma^2}}\cdot I_o\left(\frac{q_1\sqrt{m_{11}}}{\sigma^2}\right) \qquad q_1 \geq 0$$

so that

$$p_{Q_1}(q_1|H_1) = \int_0^{2\pi} p_{Q_1|H_1,\theta}(q_1|H_1,\theta)\cdot p_{\Theta}(\theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{q_1}{\sigma^2}\cdot e^{-\frac{q_1^2+m_{11}}{2\,\sigma^2}}\cdot I_o\left(\frac{q_1\sqrt{m_{11}}}{\sigma^2}\right)\cdot d\theta \qquad (3.10)$$

where the dependence on θ is imbedded in the term m_{11}. On the other hand, the output q_0 of the other envelope detector (Figure 5.5) when H_1 is assumed to be the true hypothesis, is given by

$$q_0^2 = X_0^2 + Y_0^2$$

where

$$X_0 = \int_0^T r(t)\,\sin\omega_0 t\,dt \qquad\qquad Y_0 = \int_0^T r(t)\,\cos\omega_0 t\,dt$$

Following a similar procedure to the one used above, it can be shown that

$$E\{X_0|H_1,\theta\} = \int_0^T A\cdot\sin(\omega_1 t+\theta)\cdot\sin\omega_0 t\cdot dt + \int_0^T n_j(t)\cdot\sin\omega_0 t\,dt$$

$$\equiv (AS_{1,\theta},S_0) + (n_j,S_0)$$

and

$$E\{Y_0|H_1,\theta\} = \int_0^T A\cdot\sin(\omega_1 t+\theta)\cdot\cos\omega_0 t\cdot dt + \int_0^T n_j(t)\cdot\cos\omega_0 t\ dt$$

$$\equiv (AS_{1,\theta},C_0) + (n_j,C_0)$$

Here, S_0 represents the function $\sin\omega_0 t$ and C_0 represents the function $\cos\omega_0 t$. It can be demonstrated that

$$\mathrm{Var}\{X_0|H_1,\theta\} = \mathrm{Var}\{Y_0|H_1,\theta\} = \frac{NoT}{4}$$

and also that

$$E\{\ [X_0 - E\{X_0|H_1,\theta\}]\cdot[Y_0 - E\{Y_0|H_1,\theta\}]\ |\ H_1,\theta\ \} = 0$$

so that the conditionally Gaussian random variables X_0 and Y_0 are uncorrelated, hence independent. Thus, similar to Equation (3.10), the expression for the conditional density function of q_0 becomes

$$p_{Q_0}(q_0|H_1) = \frac{1}{2\pi}\int_0^{2\pi}\frac{q_0}{\sigma^2}\cdot e^{-\frac{q_0^{\ 2}+m_{01}}{2\ \sigma^2}}\cdot I_o\left(\frac{q_0\sqrt{m_{01}}}{\sigma^2}\right)\cdot d\theta \quad (3.11)$$

where

$$m_{01} = E^2\{X_0|H_1,\theta\} + E^2\{Y_0|H_1,\theta\}$$

$$= [(AS_{1,\theta},S_0)+(n_j,S_0)]^2 + [(AS_{1,\theta},C_0)+(n_j,C_0)]^2$$

and now the dependence on θ is imbedded in the term m_{11}.

In order to compute the probability of error, we can use the previous expression for the conditional probability density functions which are derived assuming a 'mark' signal has been sent. That is, for a given value of q_1, an error is made if $q_0 > q_1$. Thus the average error probability is found by averaging the conditional error probability given by

$$P_{e1} = \Pr\{\ q_0 > q_1\ |\ H_1\ \}$$

$$= \int_0^{\infty}\left[\int_{q_1}^{\infty}(q_0|H_1)\,dq_0\right] p_{Q_1}(q_1|H_1)\,dq_1 \tag{3.12}$$

Substituting Equation (3.11) in Equation (3.12) and interchanging the order of integration, we obtain

$$P_{e1} = \frac{1}{2\pi}\int_0^{\infty}\left[\int_0^{2\pi}\left[\int_{q_1}^{\infty}\frac{q_0}{\sigma^2}\cdot e^{-\frac{q_0^2+m_{01}}{2\sigma^2}}\cdot I_o\left(\frac{q_0\sqrt{m_{01}}}{\sigma^2}\right)\cdot dq_0\right]\cdot d\theta\right.$$

$$\left.\cdot\, p_{Q_1}(q_1|H_1)\right] dq_1 \tag{3.13}$$

In the above equation, the inner-most integral can be expressed in terms of the Marcum Q function as follows

$$\int_{q_1}^{\infty}\frac{q_0}{\sigma^2}\cdot e^{-\frac{q_0^2+m_{01}}{2\sigma^2}}\cdot I_o\left(\frac{q_0\sqrt{m_{01}}}{\sigma^2}\right) dq_0 = Q\left(\frac{\sqrt{m_{01}}}{\sigma},\frac{q_1}{\sigma}\right)$$

Then, Equation (3.13) for P_{e1} with the aid of Equation (3.10) becomes

$$P_{e1} = \frac{1}{2\pi}\int_0^{\infty}\left[\int_0^{2\pi}Q\left(\frac{\sqrt{m_{01}}}{\sigma},\frac{q_1}{\sigma}\right) d\theta\right] p_{Q_1}(q_1|H_1)\,dq_1$$

$$= \int_0^{\infty}\left[\frac{1}{2\pi}\int_0^{2\pi}Q\left(\frac{\sqrt{m_{01}}}{\sigma},\frac{q_1}{\sigma}\right) d\theta\right]$$

$$\cdot\left[\frac{1}{2\pi}\int_0^{2\pi}\frac{q_1}{\sigma^2}\cdot e^{-\frac{q_1^2+m_{11}}{2\sigma^2}}\cdot I_o\left(\frac{q_1\sqrt{m_{11}}}{\sigma^2}\right) d\theta\right] dq_1 \tag{3.14}$$

From the orthogonality property of the signal pair used (which is obtained by assuming sufficient separation between

the two frequencies and that ω_1 as well as ω_0 are large), we have

$$\int_0^T \sin(\omega_1 + t)\cdot \sin\omega_0 t \cdot dt = 0$$

so that the term m_{01} is independent of θ Therefore Equation (3.14) can be rewritten in the following form

$$P_{e1} = \frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^{\infty} Q\left(\frac{\sqrt{m_{01}}}{\sigma},\frac{q_1}{\sigma}\right)\cdot\frac{q_1}{\sigma}\cdot e^{-\frac{q_1+m_{11}}{2\sigma^2}}\cdot I_0\left(\frac{q_1\sqrt{m_{11}}}{\sigma^2}\right) dq_1\right] d\theta \qquad (3.15)$$

where the order of integration has been changed. Furthermore, using the following formula involving an integral of a Marcum Q function [Ref 5],

$$\int_0^{\infty} Q\left(\frac{\alpha_2}{\sigma_2},\frac{V}{\sigma_2}\right)\cdot\frac{V}{\sigma_1^2}\cdot e^{-\frac{\alpha_1^2+V^2}{\sigma_1^2}}\cdot I_0\left(\frac{\alpha_1 V}{\sigma_1^2}\right)\cdot dV$$

$$= \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\cdot\left[1 - Q\left[\sqrt{\frac{\alpha_1^2}{\sigma_1^2+\sigma_2^2}},\sqrt{\frac{\alpha_2^2}{\sigma_1^2+\sigma_2^2}}\right]\right.$$

$$\left. + \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\cdot Q\left[\sqrt{\frac{\alpha_2^2}{\sigma_1^2+\sigma_2^2}},\sqrt{\frac{\alpha_1^2}{\sigma_1^2+\sigma_2^2}}\right]\right]$$

the inner integral in Equation (3.15) can be simplified in such a way that P_{e1} becomes

$$P_{e1} = \frac{1}{2}\left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\frac{\sqrt{m_{11}}}{\sqrt{2}\,\sigma},\frac{\sqrt{m_{01}}}{\sqrt{2}\,\sigma}\right) d\theta\right.$$

$$+ \frac{1}{2\pi}\int_{0}^{2\pi} Q\left(\frac{\sqrt{m_{01}}}{\sqrt{2}\,\sigma}, \frac{\sqrt{m_{11}}}{\sqrt{2}\,\sigma}\right) d\theta \Bigg] \tag{3.16}$$

where the dependence on θ is imbedded in the term m_{11} only.

Following exactly the same procedure used in obtaining the expression for P_{e1}, it can be established that the expression for P_{e0} which denotes the error probability when H_0 is assumed to be the true hypothesis, that is, $Pr(q_1 > q_0 \mid H_0)$ takes the form

$$P_{e0} = \frac{1}{2}\Bigg[1 - \frac{1}{2\pi}\int_{0}^{2\pi} Q\left(\frac{\sqrt{m_{00}}}{\sqrt{2}\,\sigma}, \frac{\sqrt{m_{10}}}{\sqrt{2}\,\sigma}\right) d\phi$$

$$+ \frac{1}{2\pi}\int_{0}^{2\pi} Q\left(\frac{\sqrt{m_{10}}}{\sqrt{2}\,\sigma}, \frac{\sqrt{m_{00}}}{\sqrt{2}\,\sigma}\right) d\phi \Bigg] \tag{3.17}$$

where only the term m_{00} is dependent on ϕ . Therefore the total average Probability of error P_e can be obtained from

$$P_e = \frac{1}{2}\left[P_{e1} + P_{e0} \right]$$

assuming, as previously indicated, that the two hypotheses are equally likely. Using Equation (3.16) and Equation (3.17), it is obtained that

$$P_e = \frac{1}{2}\Bigg[1 - \frac{1}{4\pi}\int_{0}^{2\pi} \left[Q\left(\frac{\sqrt{m_{11}}}{\sqrt{2}\,\sigma}, \frac{\sqrt{m_{01}}}{\sqrt{2}\,\sigma}\right) - Q\left(\frac{\sqrt{m_{01}}}{\sqrt{2}\,\sigma}, \frac{\sqrt{m_{11}}}{\sqrt{2}\,\sigma}\right)\right] d\theta$$

$$- \frac{1}{4\pi}\int_{0}^{2\pi} \left[Q\left(\frac{\sqrt{m_{00}}}{\sqrt{2}\,\sigma}, \frac{\sqrt{m_{10}}}{\sqrt{2}\,\sigma}\right) - Q\left(\frac{\sqrt{m_{10}}}{\sqrt{2}\,\sigma}, \frac{\sqrt{m_{00}}}{\sqrt{2}\,\sigma}\right)\right] d\phi \Bigg] \tag{3.18}$$

If we now use the jamming waveform which is optimum under energy constraint, when coherent FSK reception taken place that is

$$n_j(t) = \sqrt{\frac{P_{nj}}{T}} \cdot [\sin\omega_1 t - \sin\omega_0 t] \tag{3.19}$$

then the terms $m_{ij}(i,j=0,1)$ in Equation (3.18) which are a function of the jammer waveform $n_j(t)$, can be computed as follows

$$\begin{aligned} m_{11} &= [(AS_{1,\theta},S_1) + (n_j,S_1)]^2 + [(AS_{1,\theta},C_1) + (n_j,C_1)]^2 \\ &= \left(\frac{AT}{2}\right)^2 + \frac{AT\sqrt{P_{nj}}}{2}\cdot\cos\theta + \frac{P_{nj}T}{4} \end{aligned} \tag{3.20}$$

$$\begin{aligned} m_{01} &= [(AS_{1,\theta},S_0) + (n_j,S_0)]^2 + [(AS_{1,\theta},C_0) + (n_j,C_0)]^2 \\ &= \frac{P_{nj}T}{4} \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} m_{00} &= [(AS_{0,\theta},S_0) + (n_j,S_0)]^2 + [(AS_{0,\theta},C_0) + (n_j,C_0)]^2 \\ &= \left(\frac{AT}{2}\right)^2 + \frac{AT\sqrt{P_{nj}}}{2}\cdot\cos\theta + \frac{P_{nj}T}{4} \end{aligned} \tag{3.22}$$

$$\begin{aligned} m_{10} &= [(AS_{0,\theta},S_1) + (n_j,S_1)]^2 + [(AS_{0,\theta},C_1) + (n_j,C_1)]^2 \\ &= \frac{P_{nj}T}{4} \end{aligned} \tag{3.23}$$

Thus the Probability of error can be expressed in terms of SNR and JSR only, as follows

$$P_e=\frac{1}{2}\left[1-\frac{1}{4\pi}\int_0^{2\pi}\left[Q\left(\frac{\sqrt{\alpha_{11}}}{\sqrt{2}},\frac{\sqrt{\alpha_{01}}}{\sqrt{2}}\right)-Q\left(\frac{\sqrt{\alpha_{01}}}{\sqrt{2}},\frac{\sqrt{\alpha_{11}}}{\sqrt{2}}\right)\right]\cdot d\theta\right.$$

$$\left.-\frac{1}{4\pi}\int_0^{2\pi}\left[Q\left(\frac{\sqrt{\alpha_{00}}}{\sqrt{2}},\frac{\sqrt{\alpha_{10}}}{\sqrt{2}}\right)-Q\left(\frac{\sqrt{\alpha_{10}}}{\sqrt{2}},\frac{\sqrt{\alpha_{00}}}{\sqrt{2}}\right)\right]\cdot d\phi\right] \quad (3.24)$$

where

$$\alpha_{11} = SNR\cdot(2 + 2\sqrt{JSR}\cdot\cos\theta + JSR)$$

$$\alpha_{01} = \alpha_{10} = SNR\cdot JSR$$

$$\alpha_{00} = SNR\cdot(2 - 2\sqrt{JSR}\cdot\cos\phi + JSR)$$

$$SNR = \frac{\frac{A^2T}{2}}{No}$$

$$JSR = \frac{P_{nj}}{E}$$

Receiver performance can now be evaluated as a function of SNR for fixed values of JSR, using Equation (3.24).

These results have been included here so that it will be possible to compare receiver performance given by Equation (3.24) with performance of a similar receiver under the assumptions of jammer miss-synchronization or frequency offsets. In every case, it has been found that a degraded jammer produces a smaller receiver Probability of error than a 'perfect' jammer. This is to be expected and the analysis of the next two sections demonstrates this.

C. ANALYSIS OF THE EFFECT OF THE JAMMING WAVEFORM HAVING RANDOM TIME OF ARRIVAL

1. ASK

In the previous section, the effect of the deterministic jamming waveform given by Equation (3.8) on the performance of the ASK receiver of Figure (5.3) was studied. Since such an ideal jammer would have to be synchronized with the signal transmission, we now develope a more realistic model in that the difference in the arrival time of the jammer waveform with respect to the signal can be characterized by a parameter τ . Thus the difference in the time of arrival between the jammer waveform and the information waveform is given by the parameter τ so that we have

$$n_j(t)_\tau = \sqrt{P_{nj}\frac{2}{T}} \cdot \sin\omega_c(t-\tau)$$

The effect of having this time difference τ will now be analyzed using the results of the previous section.

First, using Equation (3.1), for a given value of τ the conditional means of X and Y become

$$E(X|H_1,\theta)_\tau = (AS_\theta,S) + (n_j,S)_\tau = m_{X|H_1,\theta,\tau}$$

$$E(Y|H_1,\theta)_\tau = (AS_\theta,C) + (n_j,C)_\tau = m_{Y|H_1,\theta,\tau}$$

Since the conditional variance and covariance of X and Y are independent of the signal and jammer waveform, the results on the variance and covariance of X and Y derived previously in section B.1 can be used here to obtain, for any given value of τ

$$Var(X|H_1,\theta)_\tau = Var(Y|H_1,\theta)_\tau = \frac{NoT}{4}$$

The mathematical expression previously derived for P_e is useful here in order to obtain the ASK receiver performance. Some modifications however are necessary that require the computation of

$$
\begin{aligned}
(n_j,S)_\tau &= \int_0^T \sqrt{P_{nj}\frac{2}{T}} \cdot \sin\omega_c(t-\tau)\cdot \sin\omega_c t\cdot dt \\
&= \sqrt{P_{nj}\frac{T}{2}}\cdot \left[\cos\omega_c\tau - \frac{2\cdot\sin\omega_c T\cdot\cos(\omega_c T-\omega_c\tau)}{2\cdot\sin\omega_c T} \right] \\
&= \sqrt{P_{nj}\frac{T}{2}} \cdot\cos\omega_c\tau
\end{aligned}
$$

Since $\omega_c T=n\pi$, the second term in the above bracket is zero. Similarly, we can show that

$$
(n_j,C)_\tau = \sqrt{P_{nj}\frac{T}{2}} \cdot\sin(-\omega_c\tau)
$$

These results are used in the evaluation of Equation (3.2), namely

$$
\begin{aligned}
M_\tau &= E^2\{X_1|H_1,\theta\}_\tau + E^2\{Y_1|H_1,\theta\}_\tau \\
&= [(AS_\theta,S) + (n_j,S)_\tau]^2 + [(AS_\theta,C) + (n_j,C)_\tau]^2 \\
&= \left(\frac{AT}{2}\right)^2 + AT\sqrt{P_{nj}\frac{T}{2}}\cdot\cos(\omega_c\tau+\theta) + P_{nj}\frac{T}{2} \qquad (3.25)
\end{aligned}
$$

and in the evaluation of Equation (3.3), to yield

$$
\begin{aligned}
M'_\tau &= (n_j,S)_\tau^2 + (n_j,C)_\tau^2 \\
&= P_{nj}\frac{T}{2} \qquad (3.26)
\end{aligned}
$$

Using Equation (3.7) along with the results of Equation (3.25) and (3.26), for a given value of τ it can be seen that

$$P_e(\tau) = \frac{1}{2}\cdot\left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\frac{\sqrt{M_\tau}}{\sigma}, \frac{\eta}{\sigma}\right)\cdot d\theta + Q\left(\frac{\sqrt{M'_\tau}}{\sigma}, \frac{\eta}{\sigma}\right)\right] \quad (3.27)$$

which in turn can be expressed in terms of SNR and JSR. Since

$$\frac{M_\tau}{\sigma^2} = 2SNR + 4SNR\cdot\sqrt{JSR}\cdot COS(\omega_c\tau+\theta) + 2SNR\cdot JSR$$

and

$$\frac{M'_\tau}{\sigma^2} = 2\cdot SNR\cdot JSR$$

substituting the above in Equation (3.27), $P_e(\tau)$ becomes

$$P_e(\tau) = \frac{1}{2}\left[1 + Q\left(\sqrt{2SNR\cdot JSR}, \frac{\eta}{\sigma}\right) - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{2SNR(1+2\sqrt{JSR}\cdot\cos(\omega_c\tau+\theta)+JSR)}, \frac{\eta}{\sigma}\right)\cdot d\theta\right] \quad (3.28)$$

Observe now that the integration of Equation (3.28) involving the variable θ is over the range $[0,2\pi]$. Thus regardless of the value taken by the term $\omega_c\tau$, since cosine is periodic, the values taken on by $\cos(\omega_c\tau + \theta)$ are the same as the values taken on by $\cos\theta$, with $0\le\theta\le2\pi$. Thus, Equation (3.28) is in essence identical to Equation (3.9) which yields P_e when the jammer waveform has no timing difference τ. Therefore it is clear that any timing error associated with the jammer waveform doesn't affect the performance of the incoherent receivers for ASK, when the jammer waveform is given by Equation (3.8).

2. BFSK

In this section, an analysis similar to the one carried out in the previous section is now applied to FSK modulation. The timing error associated with the jammer waveform for the FSK system is modeled by modifying Equation (3.19) as follows

$$n_j(t)_\tau = \sqrt{P_{nj}\frac{1}{T}}\cdot[\sin\omega_1(t-\tau) - \sin\omega_0(t-\tau)] \qquad (3.29)$$

As in the previous section, it is found that changing the jammer model requires only a recomputation of the terms m_{11}, m_{01}, m_{00}, and m_{10} given by Equations (3.20), (3.21), (3.22) and (3.23) respectively. Thus, using Equation (3.29), we will compute

$$E\{X_1|H_1,\theta\}_\tau = (AS_{1,\theta},S_1) + (n_j,S_1)_\tau = m_{X_1|H_1,\theta,\tau}$$

$$E\{Y_1|H_1,\theta\}_\tau = (AS_{1,\theta},C_1)_\tau + (n_j,C_1)_\tau = m_{Y_1|H_1,\theta,\tau}$$

It can easily be seen that

$$\mathrm{Var}\{X_1|H_1,\theta\}_\tau = \mathrm{Var}\{Y_1|H_1,\theta\}_\tau = \frac{NoT}{4}$$

so that these variances remain unchanged. Furthermore,

$$(n_j,S_1)_\tau = \int_0^T \sqrt{P_{nj}\frac{1}{T}}\cdot[\sin\omega_1(t-\tau) - \sin\omega_0(t-\tau)]\cdot\sin\omega_1 t\cdot dt$$

$$= \sqrt{P_{nj}\frac{T}{2}}\cdot\cos\omega_1\tau$$

and

$$(n_j,C_1)_\tau = \int_0^T \sqrt{P_{nj}\frac{1}{T}}\cdot[\sin\omega_1(t-\tau) - \sin\omega_0(t-\tau)]\cdot\cos\omega_1 t\cdot dt$$

$$= \sqrt{P_{nj}\frac{T}{2}}\cdot\sin(-\omega_1\tau)$$

Therefore, from Equation (3.20), we obtain

$$m_{11\tau} = [(AS_{1,\theta},S_1) + (n_j,S_1)_\tau]^2 + [(AS_{1,\theta},C_1) + (n_j,C_1)_\tau]^2$$
$$= \left(\frac{AT}{2}\right)^2 + \frac{AT\sqrt{P_{nj}}}{2}\cdot\cos(\omega_1\tau+\theta) + \frac{P_{nj}T}{4}$$

and from Equation (3.21), we obtain

$$m_{01\tau} = [(AS_{1,\theta},S_0) + (n_j,S_0)_\tau]^2 + [(AS_{1,\theta},C_0) + (n_j,C_0)_\tau]^2$$
$$= \frac{P_{nj}T}{4}$$

Also from Equation (3.22), we have

$$m_{00\tau} = [(AS_{0,\theta},S_0) + (n_j,S_0)_\tau]^2 + [(AS_{0,\theta},C_0) + (n_j,C_0)_\tau]^2$$
$$= \left(\frac{AT}{2}\right)^2 + \frac{AT\sqrt{P_{nj}}}{2}\cdot\cos(\omega_0\tau+\theta) + \frac{P_{nj}T}{4}$$

and from Equation (3.23), we have

$$m_{10\tau} = [(AS_{0,\theta},S_1) + (n_j,S_1)_\tau]^2 + [(AS_{0,\theta},C_1) + (n_j,C_1)_\tau]^2$$
$$= \frac{P_{nj}T}{4}$$

By substituting the above results in Equation (3.24) we obtain P_e for a given value of τ namely

$$P_e=\frac{1}{2}\left[1 - \frac{1}{4\pi}\int_0^{2\pi}\left[Q\left(\frac{\sqrt{\alpha_{11}}}{\sqrt{2}},\frac{\sqrt{\alpha_{01}}}{\sqrt{2}}\right) - Q\left(\frac{\sqrt{\alpha_{01}}}{\sqrt{2}},\frac{\sqrt{\alpha_{11}}}{\sqrt{2}}\right)\right]\cdot d\theta\right.$$
$$\left.- \frac{1}{4\pi}\int_0^{2\pi}\left[Q\left(\frac{\sqrt{\alpha_{00}}}{\sqrt{2}},\frac{\sqrt{\alpha_{10}}}{\sqrt{2}}\right) - Q\left(\frac{\sqrt{\alpha_{10}}}{\sqrt{2}},\frac{\sqrt{\alpha_{00}}}{\sqrt{2}}\right)\right]\cdot d\phi\right]$$

where

$$\alpha_{11} = SNR\cdot(2 + 2\sqrt{JSR}\cdot\cos(\omega_1\tau+\theta) + JSR)$$

$$\alpha_{01} = \alpha_{10} = SNR\cdot JSR$$

$$\alpha_{00} = SNR\cdot(2 - 2\sqrt{JSR}\cdot\cos(\omega_0\tau+\phi) + JSR)$$

$$SNR = \frac{\frac{A^2T}{2}}{No}$$

$$JSR = \frac{P_{nj}}{E}$$

By observing above probability error equation it is noted the the only term changed is $\cos\theta$. Although the $\cos\theta$ term is changed, due to the integration of the θ over interval $[0,2\pi]$, whatever the value τ is, the P_e doesn't change. The timing error associated with the jammer waveform does not affect the performance of the incoherent receiver.

An observation similar to the one made at the end of the previous section reveals here also that $P_e(\tau)$ is actually independent of τ . Therefore, timing errors associated with the jammer of Equation (3.19) do not affect in anyway the performance of the receiver analyzed in this section.

D. ANALYSIS ON THE EFFECT OF THE JAMMING WAVEFORM HAVING RANDOM FREQUENCY ERRORS

In this section, the effect of frequency errors associated with the jamming waveform is analyzed insofar as the performance of incoherent binary receivers is concerned. Results derived in section B that are applicable to the present problem will be utilized in this section. As is will be seen, frequency errors associated with the jamming waveform change only the mean value of the receiver output. This fact makes the analysis of the receiver performance more tractable.

1. ASK

In the previous section, we studied the effect of timing errors in the jamming waveform. Now we assume that the frequency error of the jammer waveform is $\Delta\omega$ while the

timing error is zero or can be shown to be negligible. Thus we have

$$n_j(t) = K \sin(\omega_c + \Delta\omega)t$$

Since the jammer power is constrained to be P_{nj}, the constant K must be consistently defined. Since we have

$$\|n_j\|^2 = \int_0^T K^2 \cdot \sin^2(\omega_c+\Delta\omega)t \cdot dt$$

$$= \frac{K^2 T}{2} \cdot \left[1 - \frac{\sin 2(\omega_c+\Delta\omega)T}{2(\omega_c+\Delta\omega)T} \right] = P_{nj}$$

we must have

$$K = \sqrt{\frac{P_{nj}}{T \cdot [K']}}$$

where

$$K' = \left[1 - \frac{\sin 2(\omega_c+\Delta\omega)T}{2(\omega_c+\Delta\omega)T} \right]$$

In order to evaluate the receiver P_e using Equation (3.1), for a given value of $\Delta\omega$, the conditional means of X and Y become

$$E\{X|H_1,\theta\}_{\Delta\omega} = (AS_\theta,S) + (n_j,S)_{\Delta\omega} = m_{X|H_1,\theta,\Delta\omega}$$

$$E\{Y|H_1,\theta\}_{\Delta\omega} = (AS_\theta,C) + (n_j,C)_{\Delta\omega} = m_{Y|H_1,\theta,\Delta\omega}$$

where S_θ, C, and S have been defined in the previous chapter. It can be shown that the variance and covariance of X and Y conditioned on H_1 and θ are independent of the signal and jammer waveform, so that in spite of the frequency error $\Delta\omega$ present, we have

$$\mathrm{Var}\{X|H_1,\theta\}_{\Delta\omega} = \mathrm{Var}\{Y|H_1,\theta\}_{\Delta\omega} = \frac{NoT}{4}$$

and

$$E\{ [X - m_{X|H_1,\theta,\Delta\omega}]\cdot[Y - m_{Y|H_1,\theta,\Delta\omega}] \;|H_1,\theta,\Delta\omega \} = 0$$

In order to compute the mean value of the receiver output under the assumption of jammer frequency error as described above, we must compute

$$(n_j,S)_{\Delta\omega} = \int_0^T K\cdot\sin(\omega_c+\Delta\omega)t\cdot\sin\omega_c t\cdot dt$$
$$= \frac{KT}{2}\cdot\left[\frac{\sin\Delta\omega T}{\Delta\omega T} - \frac{\sin(2\omega_c+\Delta\omega)T}{(2\omega_c+\Delta\omega)T} \right]$$

and

$$(n_j,C)_{\Delta\omega} = \int_0^T K\cdot\sin(\omega_c+\Delta\omega)t\cdot\cos\omega_c t\cdot dt$$
$$= \frac{KT}{2}\cdot\left[\frac{1 - \cos\Delta\omega T}{\Delta\omega T} - \frac{1 - \cos(2\omega_c+\Delta\omega)T}{(2\omega_c+\Delta\omega)T} \right]$$

With these results, by substituting in Equations (3.2) and (3.3), we have the conditional means M and M', given by $M_{\Delta\omega}$ and $M'_{\Delta\omega}$

$$M_{\Delta\omega} = \left(\frac{AT}{2}\right)^2 + AT[(n_j,S)_{\Delta\omega}\cdot\cos\theta + (n_j,C)\cdot\sin\theta] + (n_j,S)_{\Delta\omega}^{\,2} + (n_j,C)_{\Delta\omega}^{\,2}$$
$$= \left(\frac{AT}{2}\right)^2 + \frac{AT}{2}\cdot K\cdot T\cdot[B'] + 4K^2\cdot T^2\cdot[C']^2 \qquad (3.30)$$

where

$$B' = \left[\frac{\sin(\Delta\omega T-\theta)+\sin\theta}{\Delta\omega T} - \frac{\sin((2\omega_c+\Delta\omega)T + \sin\theta}{(2\omega_c+\Delta\omega)T}\right]$$

$$C' = \left[\frac{\omega_c T \cdot \sin\frac{\Delta\omega}{2}T}{\Delta\omega T((2\omega_c+\Delta\omega)T)} \right]$$

and

$$M'_{\Delta\omega} = (n_j,S)_{\Delta\omega}^2 + (n_j,C)_{\Delta\omega}^2$$
$$= 4K^2 \cdot T^2 \cdot [C']^2 \tag{3.31}$$

By substituting the above results in Equation (3.7), for a given frequency error $\Delta\omega$, the receiver error probability becomes

$$P_e(\Delta\omega) = \frac{1}{2}\left[1-\frac{1}{2\pi}\int_{0}^{2\pi} Q\left(\frac{\sqrt{M_{\Delta\omega}}}{\sigma},\frac{\eta}{\sigma}\right)d\theta + Q\left(\frac{\sqrt{M'_{\Delta\omega}}}{\sigma},\frac{\eta}{\sigma}\right)\right] \tag{3.32}$$

Equation (3.32) must to be expressed in terms of SNR and JSR. Since it can be shown that

$$\frac{M_{\Delta\omega}}{\sigma^2} = 2SNR + 4SNR \cdot \sqrt{JSR}\frac{[B']}{[K']^{1/2}} + 32 \cdot SNR \cdot JSR \frac{[C']^2}{[K']}$$

$$\frac{M'_{\Delta\omega}}{\sigma^2} = 32 \cdot SNR \cdot JSR \frac{[C']^2}{[K']}$$

where

$SNR = A^2T/No$

$JSR = P_{nj}/E$

Equation (3.32) now becomes

$$P_e(\Delta\omega) = \frac{1}{2} \cdot \left[1 + Q\left(\sqrt{32 \cdot SNR \cdot JSR \frac{[C']^2}{[K']}},\frac{\eta}{\sigma}\right) - \frac{1}{2\pi}\int_{0}^{2\pi} Q\left(\sqrt{2SNR+4SNR\sqrt{JSR}\frac{[B']}{[K']^{1/2}}+32SNRJSR\frac{[C']^2}{[K']}},\frac{\eta}{\sigma}\right)d\theta\right] \tag{3.33}$$

where the dependence on $\Delta\omega$ is imbedded in the terms B',C' and K'.

By defining a normalized frequency error, $\Delta\omega_N$, the above results can be rewritten as a function of $\Delta\omega_N$, where

$$\Delta\omega_N = \frac{\Delta\omega}{\omega_c}$$

The terms B', C' and K' as a function of $\Delta\omega_N$ become

$$B' = \left[\frac{\sin(\omega_c T\cdot\Delta\omega_N-\theta)+\sin\theta}{(\omega_c T\cdot\Delta\omega_N)} - \frac{\sin(\omega_c T(2+\Delta\omega_N) + \sin\theta}{\omega_c T(2+\Delta\omega_N)}\right]$$

$$C' = \left[\frac{\sin\omega_c T\cdot\frac{\Delta\omega_N}{2}}{\Delta\omega_N\cdot\omega_c T(2+\Delta\omega_N)}\right]$$

$$K' = \left[1 - \frac{\sin 2\omega_c T(1+\Delta\omega_N)}{2\omega_c T(1+\Delta\omega_N)}\right]$$

so that

$$P_e(\Delta\omega_N) = \frac{1}{2}\cdot\left[1 + Q\left(\sqrt{32\cdot SNR\cdot JSR\,\frac{[C']^2}{[K']}},\frac{\eta}{\sigma}\right)\right.$$

$$\left. -\frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{2SNR+4SNR\sqrt{JSR}\frac{[B']}{[K']^{1/2}}+32SNRJSR\frac{[C']^2}{[K']}},\frac{\eta}{\sigma}\right)d\theta\right] \quad (3.34)$$

The developed expression is actually the probability of error conditioned on a given frequency error. A procedure similar to the one used to analyze the effect of frequency errors on a coherent receiver is used here in order to obtain the average probability of error for the incoherent receiver of Figure (5.3). It is clear that the conditional probability of error for a given value of $\Delta\omega_N$ is a function of $\Delta\omega_N$. Therefore, in order to compute the average value of P_e, it is assumed that $\Delta\omega_N$ can be modeled as a random variable whose probability density function is uniform over

some range $(\Delta\omega_{NL}, \Delta\omega_{NH})$, where $\Delta\omega_{NL}$ is some the lower limit and $\Delta\omega_{NH}$ is some upper limit of the normalized frequency error. With this assumption, the average P_e can be obtained from

$$P_e = \frac{1}{\Delta\omega_{N,H} - \Delta\omega_{N,L}} \int_{\Delta\omega_{N,L}}^{\Delta\omega_{N,H}} P_e(\Delta\omega_N) \cdot d\Delta\omega_N$$

or equivalently

$$P_e = \frac{1}{\Delta\omega_{N,H} - \Delta\omega_{N,L}} \int_{\Delta\omega_{N,L}}^{\Delta\omega_{N,H}} \frac{1}{2} \cdot \Bigg[1 + Q\Big(\sqrt{32 \cdot SNR \cdot JSR \frac{[C']^2}{[K']}}, \frac{\eta}{\sigma} \Big)$$
$$- \frac{1}{2\pi} \int_{0}^{2\pi} Q\Big(\sqrt{2SNR\Big(1 + 2\sqrt{JSR} \frac{[B']}{[K']^{1/2}} + 16 \cdot JSR \frac{[C']^2}{[K']}}, \frac{\eta}{\sigma} \Big) d\theta \Bigg] \cdot d\Delta\omega_N \qquad (3.35)$$

where B',C' and K' have been previously defined. The computation of P_e is complicated by the fact that B',C' and K' are themselves function of $\Delta\omega_N$.

2. BFSK

In the previous sections, a deterministic waveform has been used to model a jammer signal. Here, we introduce a more realistic jammer model which includes a frequency error. The reason for choosing such a model stems from the fact that in practical situations, a jammer will lack exact knowledge of the transmitting frequencies. Thus, for BFSK, we model the jammer as

$$n_j(t)_{\Delta\omega_1, \Delta\omega_0} = K \cdot [\sin(\omega_1 + \Delta\omega_1)t - \sin(\omega_0 + \Delta\omega_0)t]$$

Due to the power constraint on the jamming signal, we must satisfy

$$\|n_j(t)\|^2_{\Delta\omega_1,\Delta\omega_\emptyset} = \int_\emptyset^T K^2\cdot[\sin(\omega_1+\Delta\omega_1)t - \sin(\omega_\emptyset+\Delta\omega_\emptyset)]^2\cdot dt$$

$$= \frac{K^2T}{2}\cdot\left[2 - \frac{\sin 2(\omega_1+\Delta\omega_1)T}{2(\omega_1+\Delta\omega_1)T} - \frac{\sin 2(\omega_\emptyset+\Delta\omega_\emptyset)T}{2(\omega_\emptyset+\Delta\omega_\emptyset)T} - 2\cdot\frac{\sin(\omega_d+\Delta\omega_d)T}{(\omega_d+\Delta\omega_d)T} + 2\cdot\frac{\sin(\omega_s+\Delta\omega_s)T}{(\omega_s+\Delta\omega_s)T}\right] = P_{nj}$$

This means that the jammer waveform must be reexpressed as

$$n_j(t)_{\Delta\omega_1,\Delta\omega_\emptyset} = \frac{\sqrt{P_{nj}\frac{2}{T}}}{[K'']^{\frac{1}{2}}}\cdot[\sin(\omega_1+\Delta\omega_1)t - \sin(\omega_\emptyset+\Delta\omega)t]$$

where

$$K'' = \left[\ 2 - \operatorname{sinc}2(\omega_1+\Delta\omega_1)T - \operatorname{sinc}2(\omega_\emptyset+\Delta\omega_\emptyset)T - 2\operatorname{sinc}(\omega_d+\Delta\omega_d)T + 2\operatorname{sinc}(\omega_s+\Delta\omega_s)T\ \right] \tag{3.36}$$

and

$$\operatorname{sinc} X = \frac{\sin X}{X}$$

$$\omega_s = \omega_1+\omega_\emptyset \qquad \Delta\omega_s = \Delta\omega_1+\Delta\omega_\emptyset$$

$$\omega_d = \omega_1-\omega_\emptyset \qquad \Delta\omega_d = \Delta\omega_1-\Delta\omega_\emptyset$$

By assuming that the jammer waveform contains frequency errors, it turns out that the only changed statistical parameters are the mean values of the receiver output conditioned on a given frequency error. Thus we can use Equation (3.18) in order to compute P_e as a function of the frequency error. We need to compute

$$(n_j,S_1)_{\Delta\omega_1,\Delta\omega_\emptyset} = \int_\emptyset^T \left[\left[\sqrt{P_{nj}\frac{2}{T}}\cdot[K'']^{-\frac{1}{2}}\cdot(\sin(\omega_1+\Delta\omega_1)t - \sin(\omega_\emptyset+\Delta\omega_\emptyset)t)\right]\cdot\sin\omega_1 t\right]dt$$

$$= \sqrt{P_{nj}\ \frac{2}{T}}\cdot[K'']^{-\frac{1}{2}}\cdot[B''] \qquad (3.37)$$

where

$$B'' = \frac{\sin\Delta\omega_1 T}{\Delta\omega_1 T} - \frac{\sin(2\omega_1+\Delta\omega_1)T}{(2\omega_1+\Delta\omega_1)T} - \frac{\sin(\omega_d-\Delta\omega_0)T}{(\omega_d-\omega_0)T} + \frac{\sin(\omega_s+\Delta\omega_0)T}{(\omega_s+\Delta\omega_0)T} \qquad (3.38)$$

Similarly, we can show that

$$(n_j, C_1)_{\Delta\omega_1,\Delta\omega_0} = \sqrt{P_{nj}\ \frac{T}{2}}\cdot[K'']^{-\frac{1}{2}}\cdot[C''] \qquad (3.39)$$

where

$$C'' = \frac{1-\cos(2\omega_1+\Delta\omega_1)T}{(2\omega_1+\Delta\omega_1)T} + \frac{1-\cos\Delta\omega_1 T}{\Delta\omega_1 T} - \frac{1-\cos(\omega_s+\Delta\omega_0)T}{(\omega_s+\Delta\omega_0)T} - \frac{1-\cos(\omega_d+\Delta\omega_0)T}{(\omega_d+\Delta\omega_0)T} \qquad (3.40)$$

Also

$$(n_j, S_0)_{\Delta\omega_1,\Delta\omega_0} = \sqrt{P_{nj}\ \frac{T}{2}}\cdot[K'']\cdot[D''] \qquad (3.41)$$

where

$$D'' = -\frac{\sin\Delta\omega_0 T}{\Delta\omega_0 T} + \frac{\sin(2\omega_0+\Delta\omega_0)T}{(2\omega_0+\Delta\omega_0)T} + \frac{\sin(\omega_d+\Delta\omega_1)T}{(\omega_d+\Delta\omega_1)T} - \frac{\sin(\omega_s+\Delta\omega_1)T}{(\omega_s+\Delta\omega_1)T} \qquad (3.42)$$

Finally

$$(n_j, C_0)_{\Delta\omega_1,\Delta\omega_0} = \sqrt{P_{nj}\ \frac{T}{2}}\cdot[K'']^{-\frac{1}{2}}\cdot[F''] \qquad (3.43)$$

where

$$F'' = \frac{1-\cos(\omega_s+\Delta\omega_1)T}{(\omega_s+\Delta\omega_1)T} + \frac{1-\cos(\omega_d+\Delta\omega_1)T}{(\omega_d+\Delta\omega_1)T} + \frac{\cos(2\omega_0+\Delta\omega_0)T}{(2\omega_0+\Delta\omega_0)T} + \frac{\cos\Delta\omega_0 T - 1}{\Delta\omega_0 T} \quad (3.44)$$

Using Equations (3.20),(3.21),(3.22) and (3.23), the conditional means of either channel output conditioned on either hypothesis and conditioned on a given frequency error can now be computed. From Equation (3.20), we have

$$\begin{aligned}
m_{11_{\Delta\omega_1,\Delta\omega_0}} &= [(AS_{1,\theta},S_1) + (n_j,S_1)_{\Delta\omega_1,\Delta\omega_0}]^2 \\
&\quad + [(AS_{1,\theta},C_1) + (n_j,C_1)_{\Delta\omega_1,\Delta\omega_0}]^2 \\
&= (\frac{AT}{2})^2\cos^2\theta + 2(\frac{AT}{2})\cos\theta\cdot\sqrt{P_{nj}\frac{T}{2}}\cdot[K'']^{-\frac{1}{2}}\cdot[B''] \\
&\quad +P_{nj}\frac{T}{2}\cdot[K'']^{-1}\cdot[B'']^2 + (\frac{AT}{2})^2\sin^2\theta \\
&\quad +2(\frac{AT}{2})\sin\theta\sqrt{P_{nj}\frac{T}{2}}\cdot[K'']^{-\frac{1}{2}}\cdot[C''] + P_{nj}\frac{T}{2}\cdot[K'']^{-1}\cdot[C'']^2 \\
&= (\frac{AT}{2})^2 + AT\sqrt{P_{nj}\frac{T}{2}}\cdot[K'']^{-\frac{1}{2}}\cdot\Big[\,[B'']\cos\theta+[C'']\sin\theta\,\Big] \\
&\quad + P_{nj}\frac{T}{2}\cdot[K'']^{-\frac{1}{2}}\cdot\Big[\,[B'']^2+[C'']^2\,\Big]
\end{aligned}$$

From Equation (3.21), we have

$$\begin{aligned}
m_{01_{\Delta\omega_1,\Delta\omega_0}} &= [(AS_{1,\theta},S_0) + (n_j,S_0)_{\Delta\omega_1,\Delta\omega_0}]^2 \\
&\quad + [(AS_{1,\theta},C_0) + (n_j,C_0)_{\Delta\omega_1,\Delta\omega_0}]^2 \\
&= P_{nj}\frac{T}{2}\cdot[K'']^{-1}\cdot[D'']^2 + P_{nj}\frac{T}{2}\cdot[K'']^{-1}\cdot[F'']^2
\end{aligned}$$

Also from Equation (3.22),we have

$$m_{00_{\Delta\omega_1,\Delta\omega_0}} = [(AS_{0,\theta},S_0) + (n_j,S_0)_{\Delta\omega_1,\Delta\omega_0}]^2$$

$$+ [(AS_{0,\theta},C_0) + (n_j,C_0)_{\Delta\omega_1,\Delta\omega_0}]^2$$

$$= (\frac{AT}{2})^2 + AT\sqrt{P_{nj}\frac{T}{2}}\cdot[K"]^{-\frac{1}{2}}\cdot\left[[D"]\cos\theta+[F"]\sin\theta \right]$$

$$+ P_{nj}\frac{T}{2}\cdot[K"]^{-\frac{1}{2}}\cdot\left[[D"]^2+[F"]^2 \right]$$

and from EQuation (3.23), we have

$$m_{10_{\Delta\omega_1,\Delta\omega_0}} = [(AS_{0,\theta},S_1) + (n_j,S_1)_{\Delta\omega_1,\Delta\omega_0}]^2$$

$$+ [AS_{0,\theta},C_1) + (n_j,C_1)_{\Delta\omega_1,\Delta\omega_0}]^2$$

$$= P_{nj}\frac{T}{2}\cdot[K"]^{-1}\cdot[B"]^2 + P_{nj}\frac{T}{2}\cdot[K"]^{-1}\cdot[C"]^2$$

With SNR and JSR as is previously defined, we can rewrite the above equations specifying the conditional means, as

$$(\frac{m_{11}}{\sigma^2})_{\Delta\omega_1,\Delta\omega_0} = 2SNR\cdot\left[1 + 2\sqrt{JSR}\cdot[K"]^{-\frac{1}{2}}([B"]\cdot\cos\theta+[C"]\cdot\sin\theta) + JSR\cdot[K"]^{-1}([B"]^2+[C"]^2) \right]$$

$$(\frac{m_{01}}{\sigma^2})_{\Delta\omega_1,\Delta\omega_0} = 2SNR\cdot JSR\cdot[K"]^{-1}\cdot([D"]^2+[F"]^2)$$

$$(\frac{m_{00}}{\sigma^2})_{\Delta\omega_1,\Delta\omega_0} = 2SNR\cdot\left[1 + 2\sqrt{JSR}\cdot[K"]^{-\frac{1}{2}}([D"]\cdot\cos\theta+[F"]\cdot\sin\theta) + JSR\cdot[K"]^{-1}([D"]^2+[F"]^2) \right]$$

$$(\frac{m_{10}}{\sigma^2})_{\Delta\omega_1,\Delta\omega_0} = 2SNR\cdot JSR\cdot[K"]^{-1}\cdot([B"]^2+[C"]^2)$$

Using the above results, we obtain the conditional Probability of error by substituting in Equation (3.18) to yield

$$P_e(\Delta\omega_1,\Delta\omega_0)=\frac{1}{2}\left[1-\frac{1}{4\pi}\int_0^{2\pi}\left[Q\left(\sqrt{\frac{\alpha_{11}}{2}},\sqrt{\frac{\alpha_{01}}{2}}\right)-Q\left(\sqrt{\frac{\alpha_{01}}{2}},\sqrt{\frac{\alpha_{11}}{2}}\right)\right]d\theta\right.$$

$$\left.-\frac{1}{4\pi}\int_0^{2\pi}\left[Q\left(\sqrt{\frac{\alpha_{00}}{2}},\sqrt{\frac{\alpha_{10}}{2}}\right)-Q\left(\sqrt{\frac{\alpha_{10}}{2}},\sqrt{\frac{\alpha_{00}}{2}}\right)\right]d\theta\right] \qquad (3.45)$$

where

$$\alpha_{11} = 2SNR\cdot\Big[\ 1\ +2\sqrt{JSR}\cdot[K'']^{-\frac{1}{2}}([B'']\cdot\cos\theta+[C'']\cdot\sin\theta) + JSR\cdot[K'']^{-1}([B'']^2+[C'']^2)\ \Big] \qquad (3.46)$$

$$\alpha_{01} = 2SNR\cdot JSR\cdot[K'']^{-1}\cdot([D'']^2+[F'']^2) \qquad (3.47)$$

$$\alpha_{00} = 2SNR\cdot\Big[\ 1\ +2\sqrt{JSR}\cdot[K'']^{-\frac{1}{2}}([D'']\cdot\cos\theta+[F'']\cdot\sin\theta) + JSR\cdot[K'']^{-1}([D'']^2+[F'']^2)\ \Big] \qquad (3.48)$$

$$\alpha_{10} = 2SNR\cdot JSR\cdot[K'']^{-1}\cdot([B'']^2+[C'']^2) \qquad (3.49)$$

As in the previous chapter, normalized frequency errors $\Delta\omega_{N,1}$, and $\Delta\omega_{N,0}$ are introduced and it is assumed that these normalized frequency errors can be modeled as independent, uniformly distributed random variables over a given range. Thus, following the same procedure previously used, the average probability of error can be obtained as a function of $\Delta\omega_{N,1}$ and $\Delta\omega_{N,0}$. Using the assumption of independence and of uniform distribution of the frequency errors, we obtain

$$P_e = \frac{1}{\Delta\omega_{N,1,H} - \Delta\omega_{N,1,L}} \cdot \frac{1}{\Delta\omega_{N,0,H} - \Delta\omega_{N,0,L}} \cdot \int_{\Delta\omega_{N,1,L}}^{\Delta\omega_{N,1,H}} \int_{\Delta\omega_{N,0,L}}^{\Delta\omega_{N,0,H}}$$

$$\cdot P_e(\Delta\omega_{N,1}, \Delta\omega_{N,0}) \cdot d\Delta\omega_{N,1} \cdot d\Delta\omega_{N,0}$$

Using now Equation (3.45), we obtain

$$P_e = \frac{1}{\Delta\omega_{N,1,H} - \Delta\omega_{N,1,L}} \cdot \frac{1}{\Delta\omega_{N,0,H} - \Delta\omega_{N,0,L}} \cdot \int_{\Delta\omega_{N,1,L}}^{\Delta\omega_{N,1,H}} \int_{\Delta\omega_{N,0,L}}^{\Delta\omega_{N,0,H}}$$

$$\frac{1}{2}\left[1 - \frac{1}{4\pi}\int_0^{2\pi} \left[Q\left(\sqrt{\frac{\alpha_{11}}{2}}, \sqrt{\frac{\alpha_{01}}{2}}\right) - Q\left(\sqrt{\frac{\alpha_{01}}{2}}, \sqrt{\frac{\alpha_{11}}{2}}\right)\right] d\theta\right.$$

$$\left. - \frac{1}{4\pi}\int_0^{2\pi} \left[Q\left(\sqrt{\frac{\alpha_{00}}{2}}, \sqrt{\frac{\alpha_{10}}{2}}\right) - Q\left(\sqrt{\frac{\alpha_{10}}{2}}, \sqrt{\frac{\alpha_{00}}{2}}\right)\right] d\theta\right] d\Delta\omega_{N,1} d\Delta\omega_{N,0} \tag{3.50}$$

where α_1, α_2, α_3, and α_4 have been defined by Equations (3.46),(3.47),(3.48), and (3.49) respectively, and the parameters K",B",C" and F" have been defined by Equations (3.36),(3.38),(3.42) and (3.44), respectively.

IV. DESCRIPTION OF GRAPHICAL RESULTS

A. GENERAL

In previous chapters, mathematical models of optimum jammers were introduced in such a way as to obtain more realistic jammer models which include timing errors and frequency errors. Analyses were carried out in order to determine the effect of the timing errors or frequency errors on the performance of coherent as well as incoherent digital receivers. This work has resulted in four main equations which specify the performance of the receiver in terms of the probability of error as a function of signal to noise ratio (SNR) and jammer to signal ratio (JSR). With the aid of computer evaluations, the receiver probability of error for various jammer models has been obtained as a function of SNR for a given JSR value and the results plotted for each case considered.

In each plot, the JSR=0 case has been included in order to provide a basis for comparisons of the jammer effectiveness on the receiver performance as it relates to additive white Gaussian noise only interference. Additionally the effect of a perfect jammer(τ=0,$\Delta\omega$=0) is also evaluted in order to determine jammer degradation resulting from timing or frequency errors.

The effect of the jammer waveform on the coherent receiver will be presented first and then, the results for the incoherent receiver will be presented next.

B. COHERENT RECEIVERS

1. PSK

Graphical results on the performance of a coherent BPSK receiver in the presence of optimum jamming are presented in Figure (5.6), based on the results given by

Equation (2.7). Plots of P_e were computed as a function of SNR for fixed values of JSR.

Figure (5.6) clearly shows the 'break point' phenomenon described in [Ref 1], namely that as a JSR increases to a value of one or greater, P_e does not decrease with increasing SNR. That is, for JSR value greater than or equal to one, P_e increases to the value of 1/2 with increasing SNR. From this figure, it can be noted that 10.2 dB of SNR is required to obtain a P_e of 10^{-6} at a JSR value of 0. In comparison, it takes 14 dB of SNR to obtain the same P_e for a JSR value of 0.1.

Figure (5.7) displays the probability of error for the same coherent receiver under the assumption that the optimum jammer previously considered suffers from a timing error τ_N which is assumed to be uniformly distributed over the inteval [0,1].

It can be seen that now 13 dB of SNR is required to obtain the P_e of 10^{-6} at JSR = 0.1. We can compare this result to the 14 dB SNR required when the jammer model does not include a timing error. It is apparent however from these results that the penalty for lack of coherence on the part of the jammer with the signal transmission, is not severe. Therefore, without knowledge of correct timing for synchronization with the adverary's coherent receiver, the jammer has to increase its power by a small amount in order to produce the same error rate at the receiver when jammer synchronization is perfect.

When the jammer model incorporates frequency estimation errors, the performance of the receiver is given by Equation (2.33) and Figure (5.8) shows the resulting P_e of the receiver. The plots shown in this figure were obtained under the assumption that the normalized frequency error $\Delta\omega_N$ can be modeled as a random variable uniformly distributed over the range which in this case has been

assumed to be [-0.001,0.001] and that the number of waveform cycles in one observation period is 10. By analyzing Figure (5.8), it is noted that P_e of 10^{-6} can be obtained with 13.6 dB of SNR, when JSR is 0.1.

TABLE 1

SNR REQUIREMENTS ($P_E = 10^{-6}$ PSK)

JSR	DET' JAMM	JAMM WITH TIME ERR	JAMM WITH FREQ' ERR
JSR=0.0	10.2 dB	10.2 dB	10.2 dB
JSR=0.1	14.0 dB	13.0 dB	13.6 dB
JSR=0.5	26.0 dB	20.0 dB	20.0 dB
JSR=1.0	unable	large	46.0 dB

The values in Table 1 were obtained directly from the figures corresponding to P_e when the jammer exhibits timing errors or frequency errors.

2. <u>BFSK</u>

The procedure used in the previous section is also appropriate in the analysis of the figure presenting the results on the performance of the coherent BFSK receiver. Figure (5.9) displays the P_e of this receiver in the presence of an optimum deterministic jammer which exhibits no timing or frequency errors. One can note that when JSR = 0, 13.4 dB SNR is required in order to obtain a P_e of 10^{-6}, whereas when the JSR value increases to 0.1, 18 dB of SNR is required to maintain a P_e of 10^{-6}. Figure (5.10) shows plots of P_e for the same coherent receiver when the jamming waveform suffers from timing errors. For P_e of 10^{-6}, at JSR = 0.1, 17 dB SNR is required. This is 1 dB less SNR than

that required for the case of a jammer waveform with no timing errors. In order to obtain these results, it was assumed that the normalized timing error τ_N, can be modeled as a uniformly distributed random variable over [0,1].

Figure (5.11) displays plots of P_e for the same receiver when the jammer is assumed to contain frequency errors. This figure was obtained under the assumption that n=6, m=4 where 'n' is the number of cycles of the summ frequency ($\omega_s T=2n\pi$) and 'm' is the number of cycles of the difference frequency ($\omega_d T=2m\pi$). The normalized frequency errors $\Delta_{N,1}$ and $\Delta_{N,0}$ are modeled as statisticly independent random variables uniformly distributed over [-0.1,0.1]. The results of required SNR for a given P_e at different values of JSR are summarized in Table 2.

TABLE 2

SNR REQUIREMENTS (P_E = 10^{-6} FSK)

JSR	DET' JAMM	JAMM WITH TIME ERR	JAMM WITH FREQ' ERR
JSR=0.0	13.0 dB	13.0 dB	13.0 dB
JSR=0.1	18.0 dB	17.0 dB	17.8 dB

C. INCOHERENT RECEIVERS

In Chapter 3, section C, it was shown that the jammer model utilized which includes timing errors affects the performance of incoherent receivers in exactly the same way as a jammer with no timing errors. That is, the timing error associated with the jammer does not affect the resulting performance of the incoherent receiver. Therefore the

performance differences associated with the jamming waveforms with frequency errors and without will be analyzed here.

1. ASK

The performance of the incoherent ASK receiver is graphically displayed in Figure (5.12). These plots correspond to numerical evaluation of Equation (3.9) under the assumption of equally likely hypotheses. This figure clearly shows the 'break point' effect at JSR of 0.25. It can be observed that 16 dB of SNR is required in order to obtain a P_e of 10^{-5} when no jammer is present(that is JSR=0), while it takes 23.5 dB of SNR to obtain the same P_e for a JSR value of 0.1.

Figure (5.13) displays the performance of the same ASK receiver except that now the jammer exhibits frequency errors. Figure (5.13) was obtained under the assumption that the number of waveform cycles per observation period is 5 and the normalized frequency error $\Delta\omega_N$ can again be modeled as uniformly distributed random variable over [-0.1,0.1].

Equation (3.35) was used to generate the plots of Figure (5.13). It can be observed that at JSR=0.1 in order to obtain P_e of 10^{-5}, the value of SNR required is 23.0 dB. The key results for this case are summarized in Table 3.

2. FSK

Figure (5.14) and Figure (5.15) correspond to graphical results on the performance of the coherent FSK receiver. When the jamming waveform is modeled as deterministic, P_e has been evaluated using Equation (3.24) and graphically displayed in Figure (5.14). Here, a 'break point' occurs at a JSR value somewhere between 0.5 and 1.0 as shown on the figure. It can be noted that 13.5 dB of SNR is required in order to obtain a P_e of 10^{-5} for a JSR value of 0.0, while the same P_e is obtained by increasing the SNR to 16.5 dB for a JSR value of 0.1.

TABLE 3

SNR REQUIREMENTS (P_E = 10^{-5} ASK)

JSR	DET' JAMM	JAMM WITH TIME ERR	JAMM WITH FREQ' ERR
JSR= 0.0	16.0 dB	16.0 dB	16.0 dB
JSR= 0.1	23.5 dB	23.5 dB	23.0 dB
JSR=0.25	unable	unable	large

Figure (5.15) displays P_e plots corresponding to the same receiver under the assumption that tha jammer contains frequency errors. In this case, 16.8 dB of SNR is needed in order to obtain a P_e of 10^{-5} at a JSR value of 0.1. The key results are summarized in Table 4.

TABLE 4

SNR REQUIREMENTS (P_E = 10^{-5} ASK)

JSR	DET' JAMM	JAMM WITH TIME ERR	JAMM WITH FREQ' ERR
JSR=0.0	13.5 dB	13.5 dB	13.5 dB
JSR=0.1	16.5 dB	16.5 dB	16.8 dB
JSR=0.3	23.5 dB	23.5 dB	22.5 dB

These results were obtained under the assumption that the normalized frequency errors $\Delta\omega_{N,1}$ and $\Delta\omega_{N,0}$ are statistically independent random variables uniformly distributed over [-0.1,0.1] and n=6, m=4 while 'n' is the number of cycles of summ frequency per observed period and

'm' is the number of cycles of the difference frequency per observed period.

V. CONCLUSIONS

In this thesis, the effect of timing errors or frequency errors associated with jamming waveforms used to degrade the performance of coherent as well as incoherent receivers has been analyzed. It has been assumed that these receivers are designed to operate on a signal plus white Gaussian noise only environment.

The analysis of the effectiveness of jammers that exhibit timing and frequency errors was undertaken by evaluating the receiver probability of error in the presence of jamming, and comparing the results to those obtained when the jamming had no timing or frequency errors. Thus, receiver error probabilities as a function jammer-to-signal ratio (JSR) and signal-to-noise ratio (SNR) were evaluated and determinations were made on how much larger JSR values were required as a result of timing or frequency errors, in order to produce a given P_e at some fixed SNR value.

For the coherent receivers analyzed, the effect of the timing error associated with the jammer was studied and it was concluded that the jammer's lack of sychronism with the digital data causes a small decrease in jammer effectiveness. Typically 1 or 2 dB more JSR was needed in order to overcome timing errors. Due to the lack of correct knowledge of the frequencies of communications, the jammer required typically 1 or 2 dB more power in order to compensate for frequency errors.

For the incoherent receivers analyzed, it was determined that the timing errors associated with the jamming waveform have no effect on the performance of the receiver. However, the jammer's incorrect knowledge of the communication system's operating frequencies results in a loss of jammer effectiveness to the point that jammer power must be

incresed by about 1 to 2 dB in order to overcome the lack of precise knowledge of the communication system's operating frequencies.

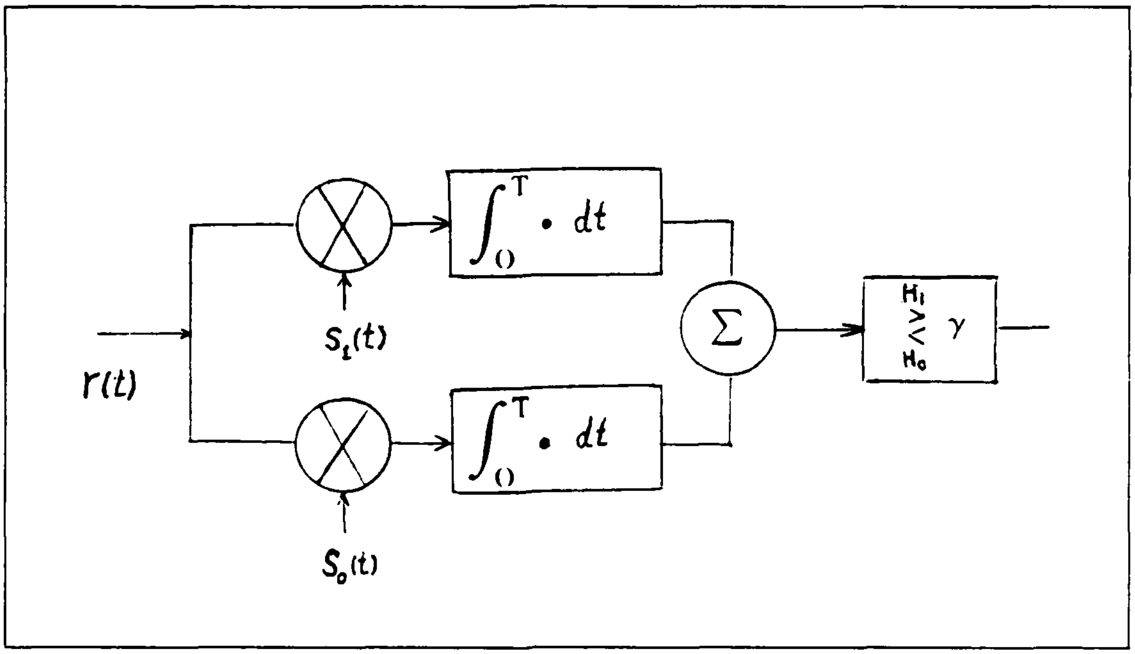

Figure 5.1 Coherent Correlator Receiver

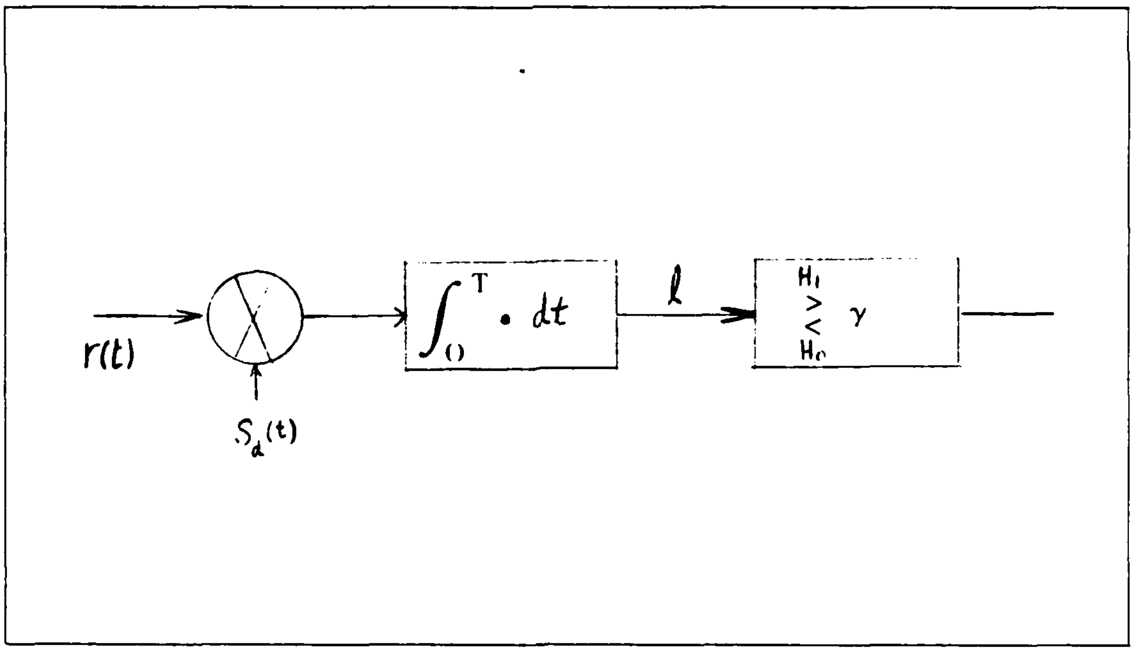

Figure 5.2 Single Correlator Coherent Receiver

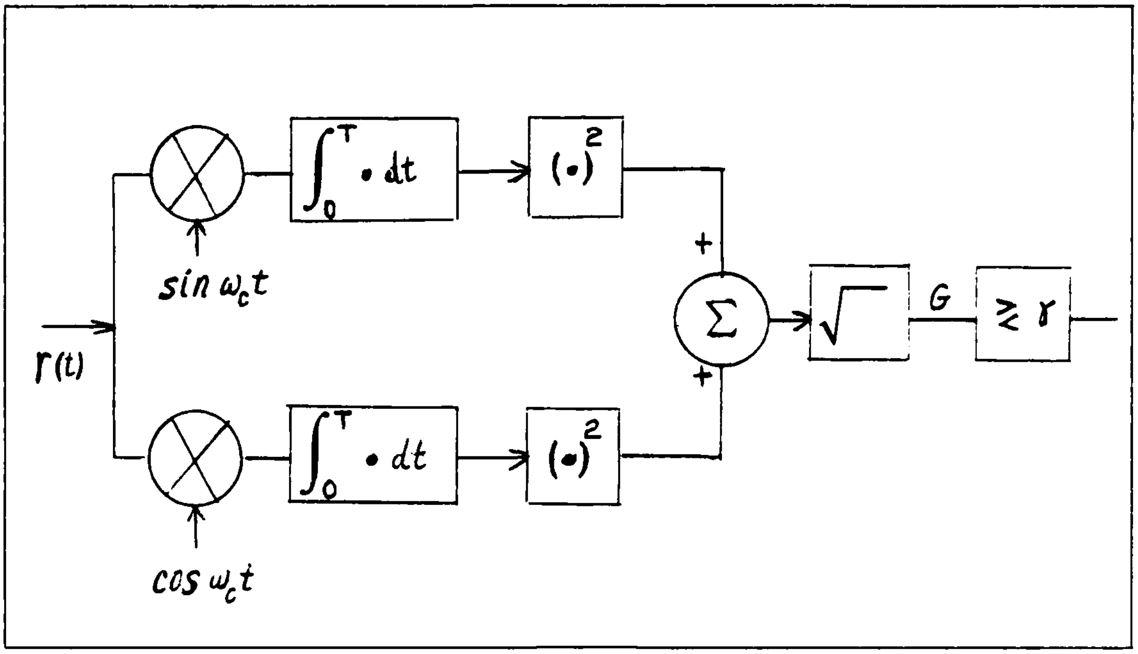

Figure 5.3 Quadrature Incoherent Receiver for ASK

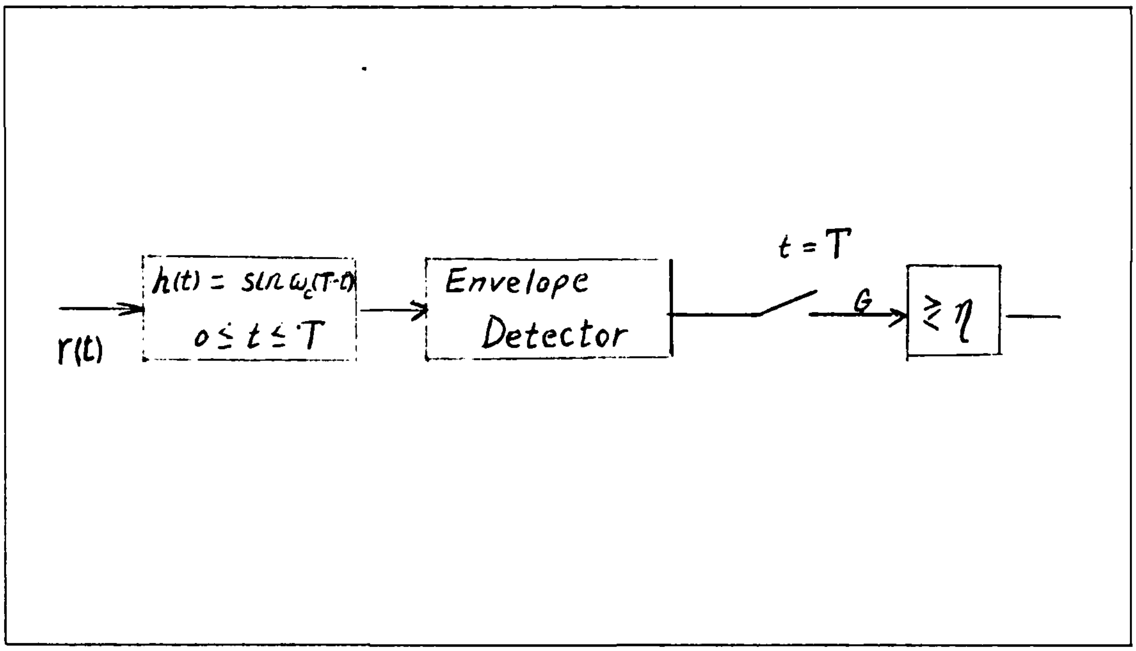

Figure 5.4 Incoherent Matched Filter Receiver

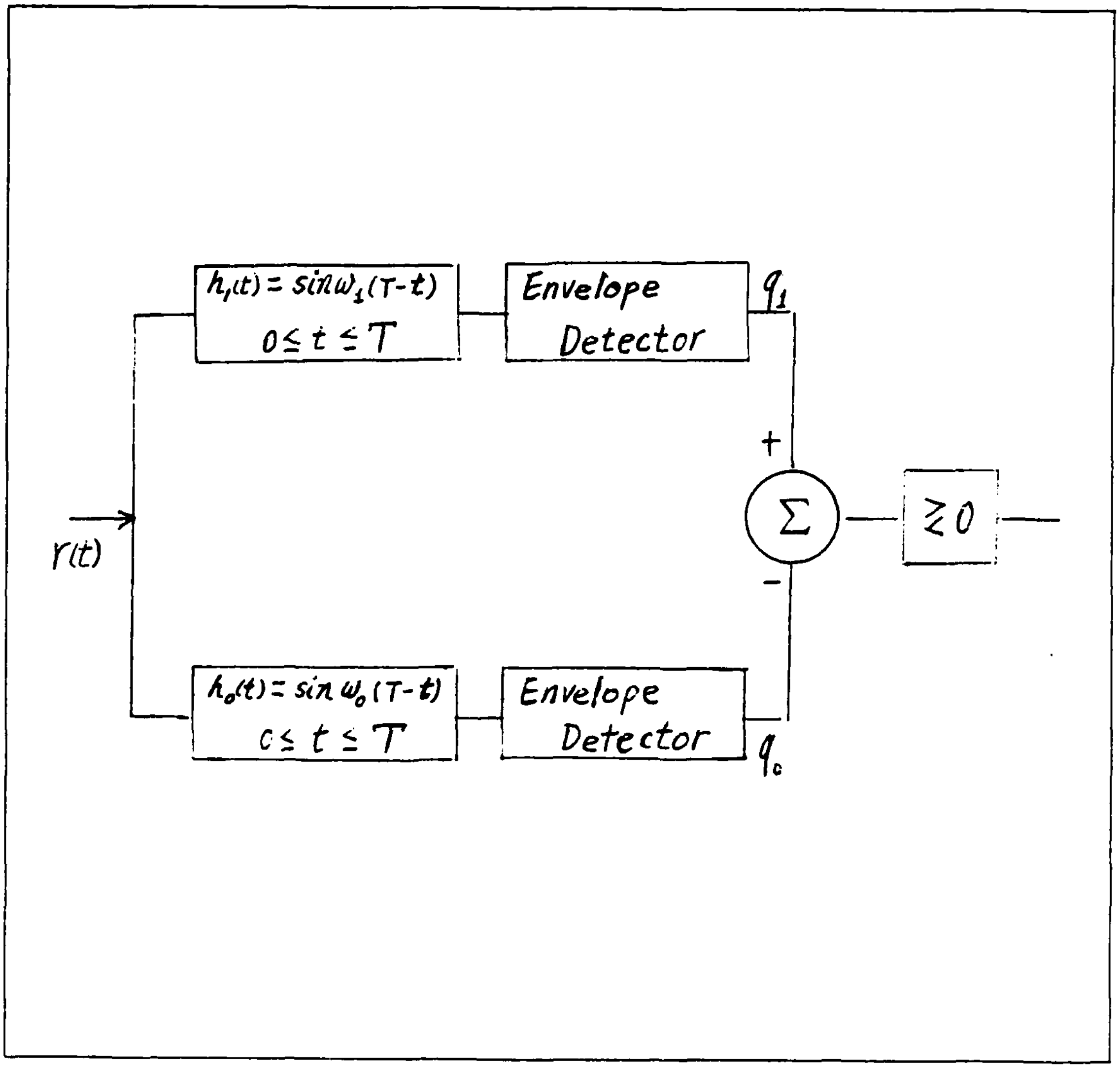

Figure 5.5 Incoherent receiver for FSK

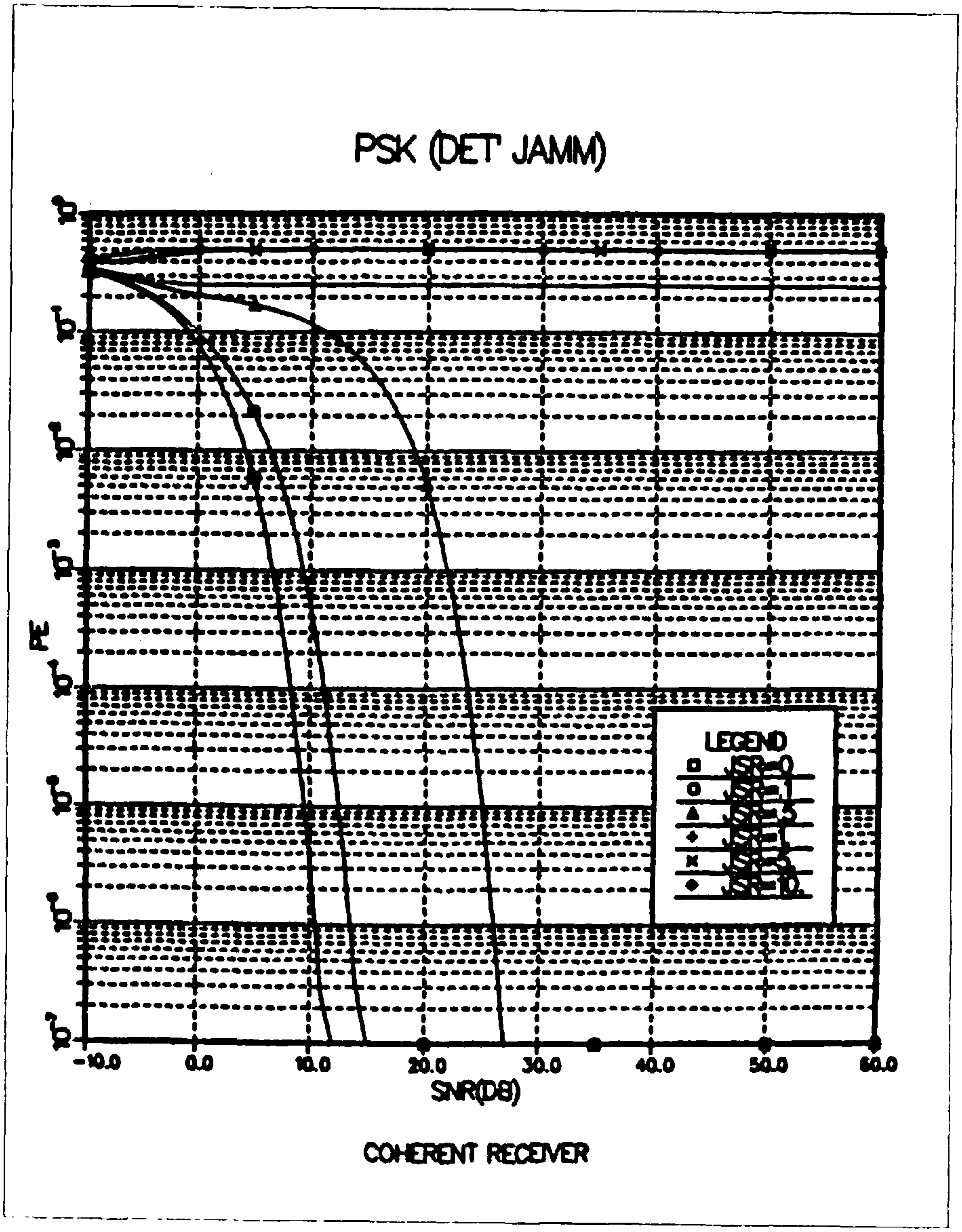

Figure 5.6 Performance of the PSK Coherent Receiver with Deterministic Jamming

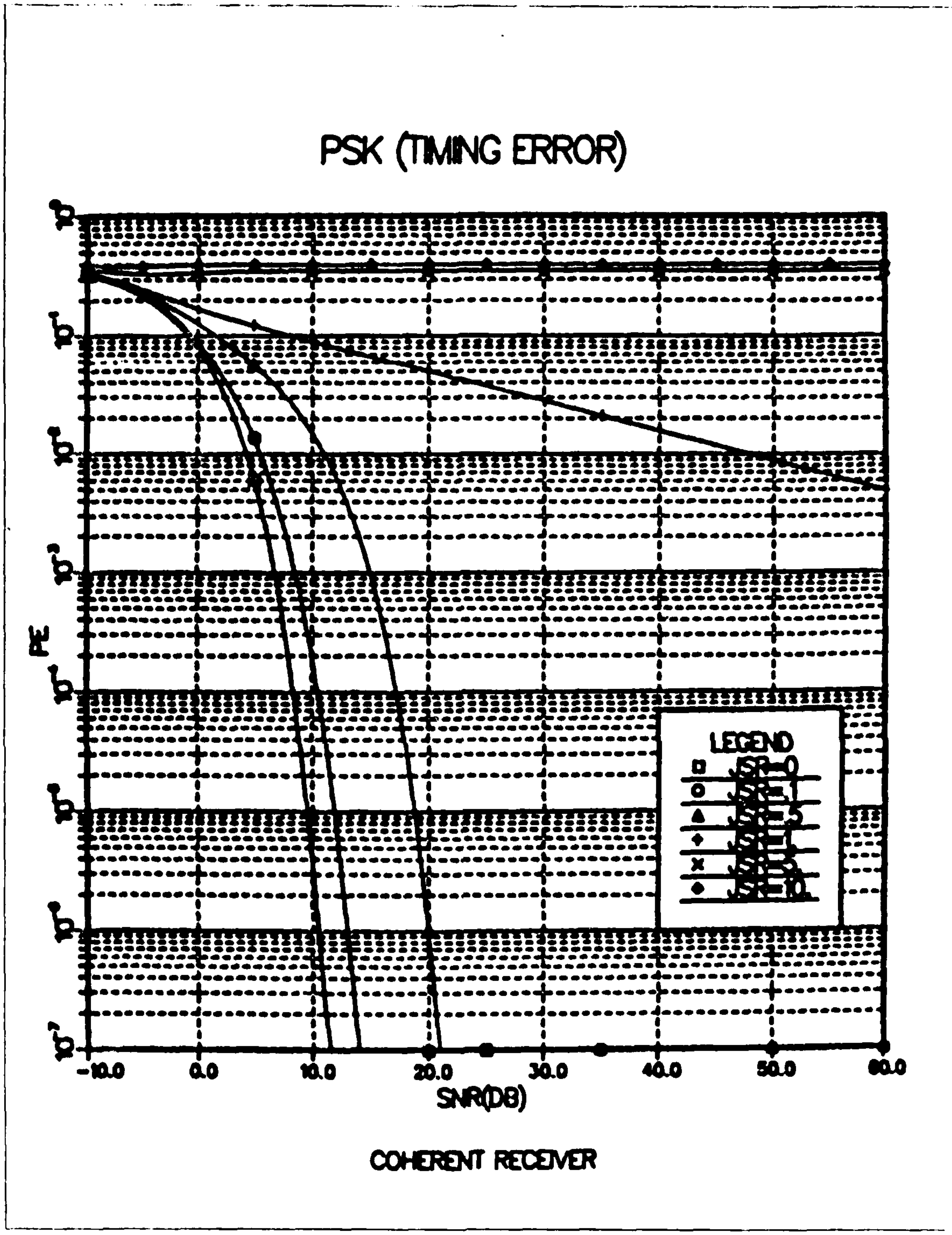

Figure 5.7 Performance of the PSK Coherent Receiver with Timing Error Jamming

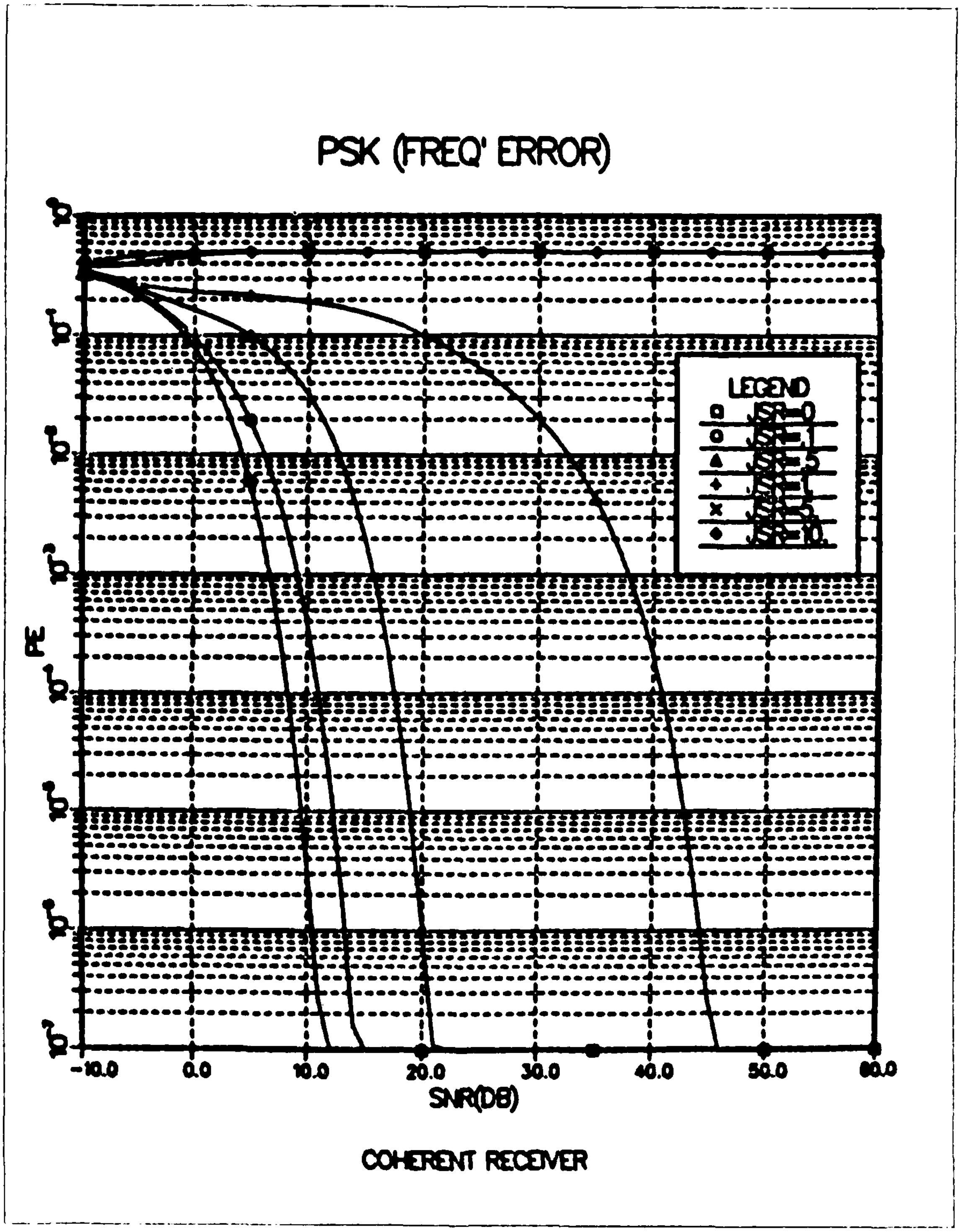

Figure 5.8 Performance of the PSK Coherent Receiver with Frequency Error Jamming

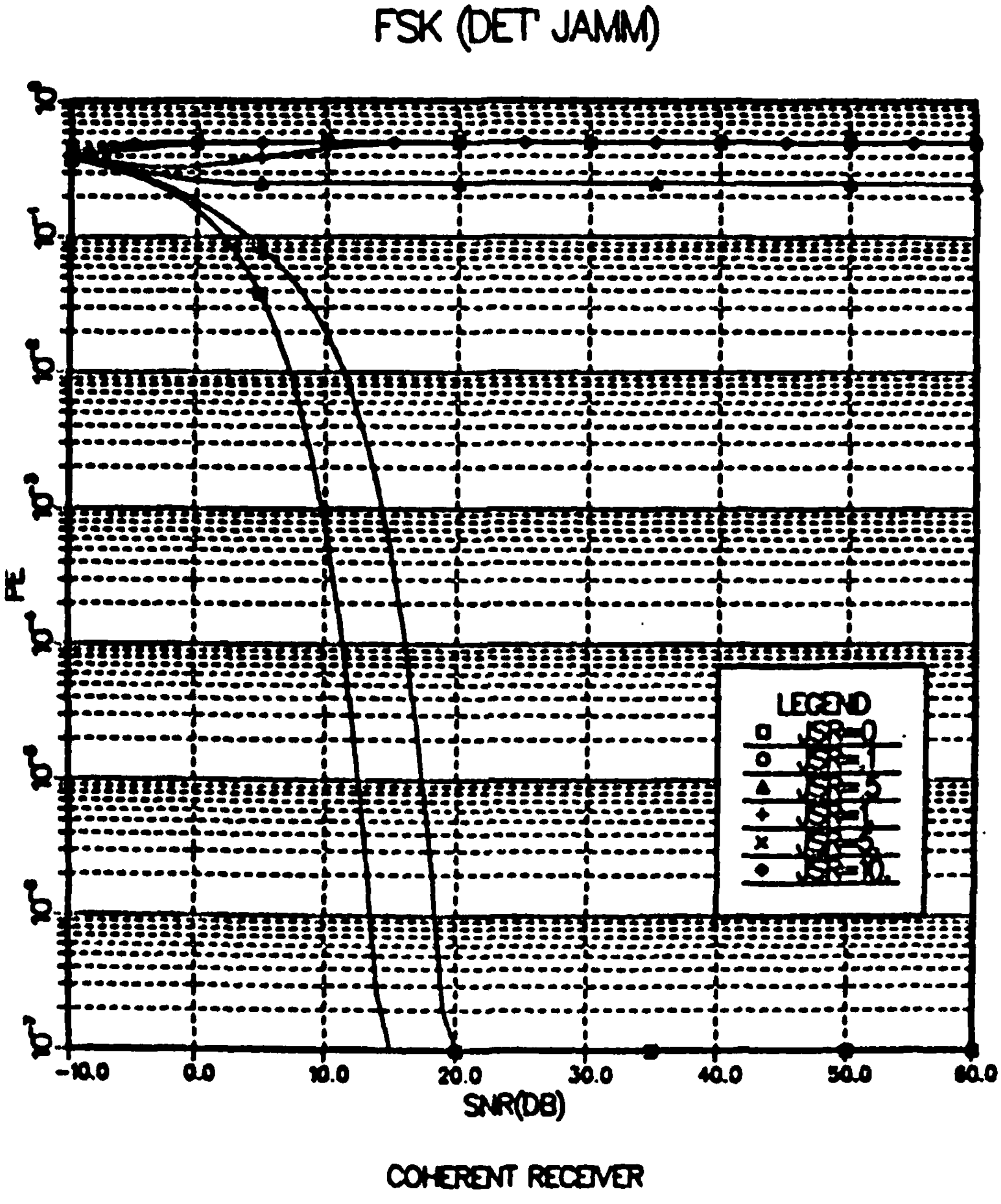

Figure 5.9 Performance of the FSK Coherent Receiver with Deterministic Jamming

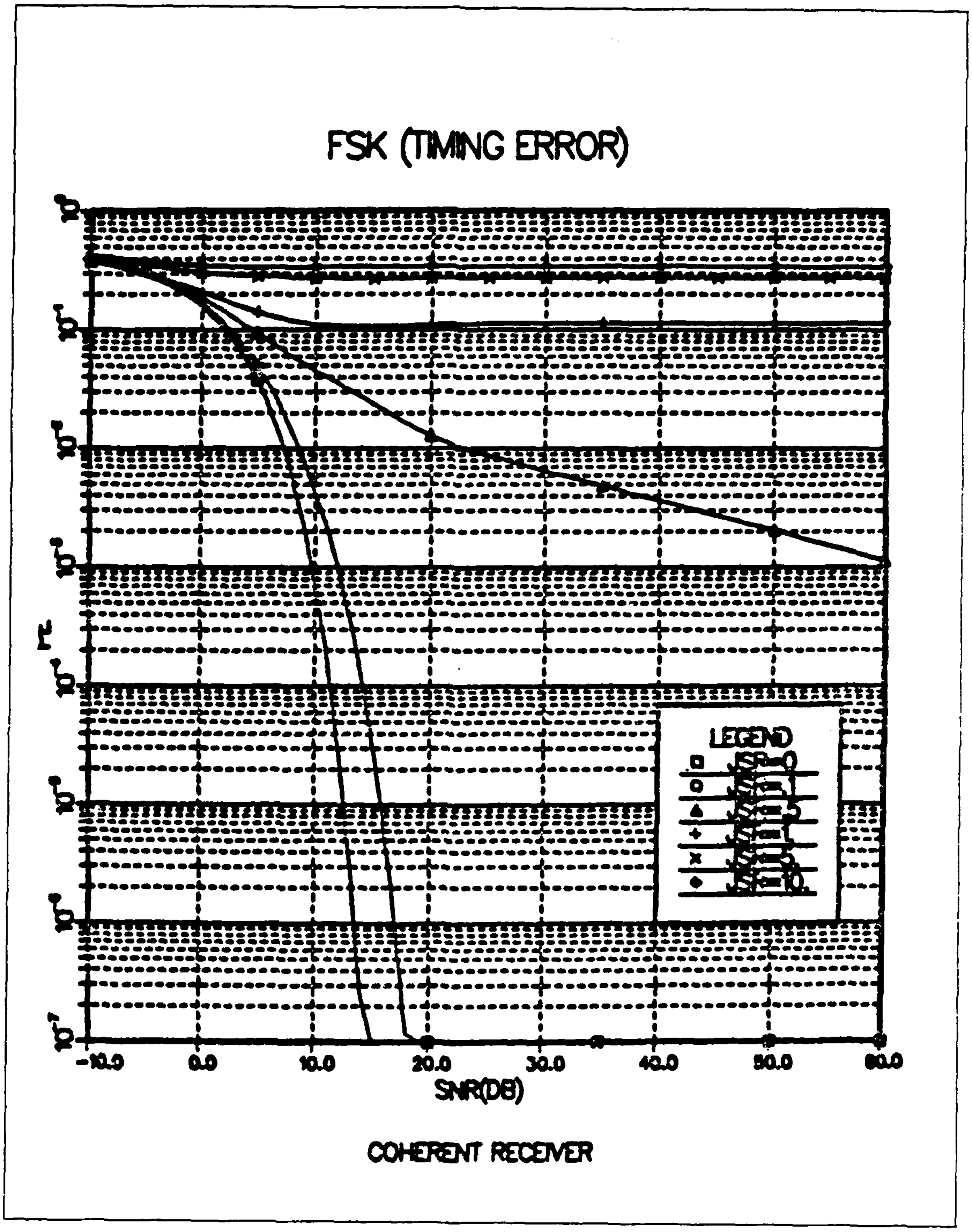

Figure 5.10 Performance of the FSK Coherent Receiver with Timing Error Jamming

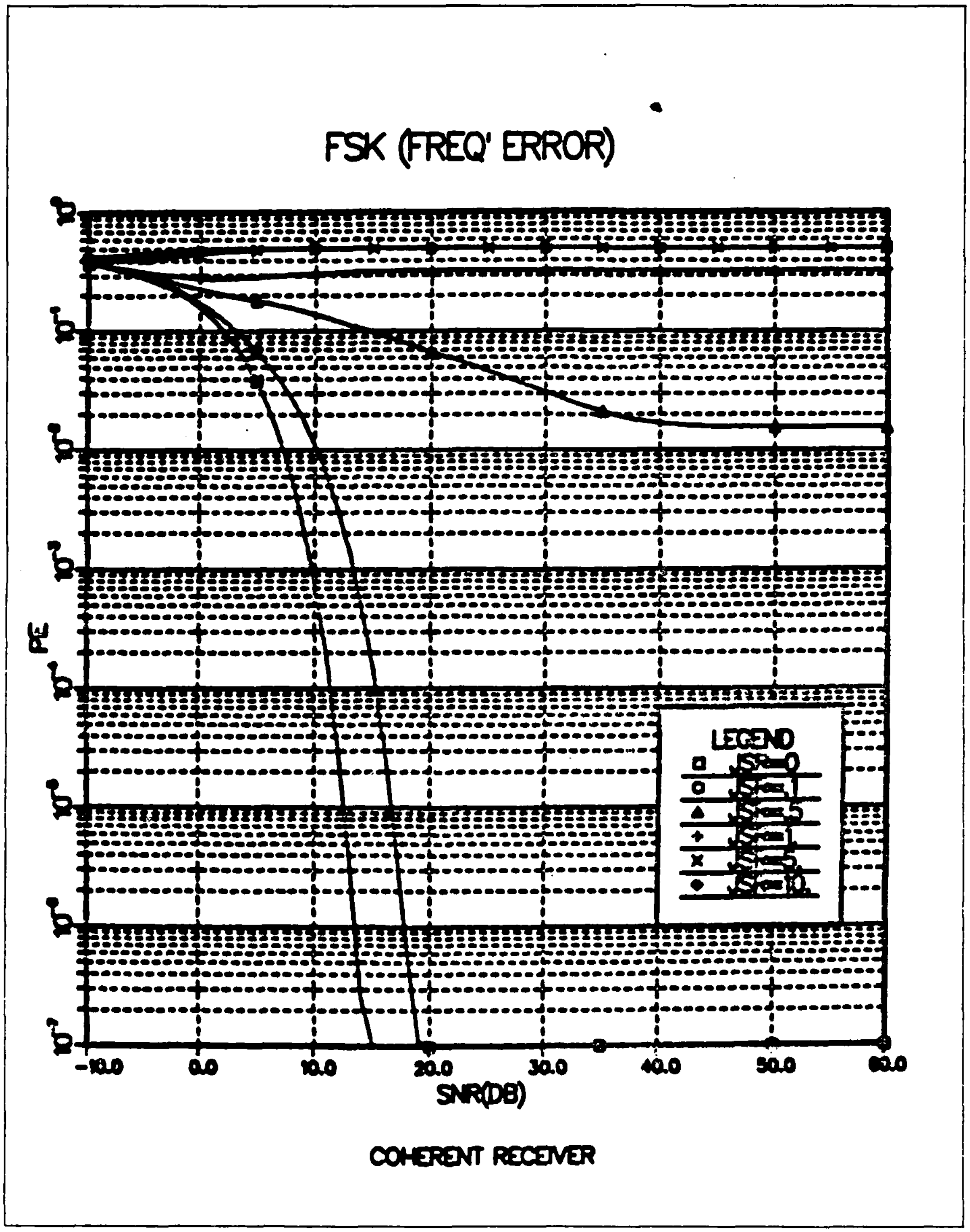

Figure 5.11 Performance of the FSK Coherent RECEIVER with Frequency Error Jamming

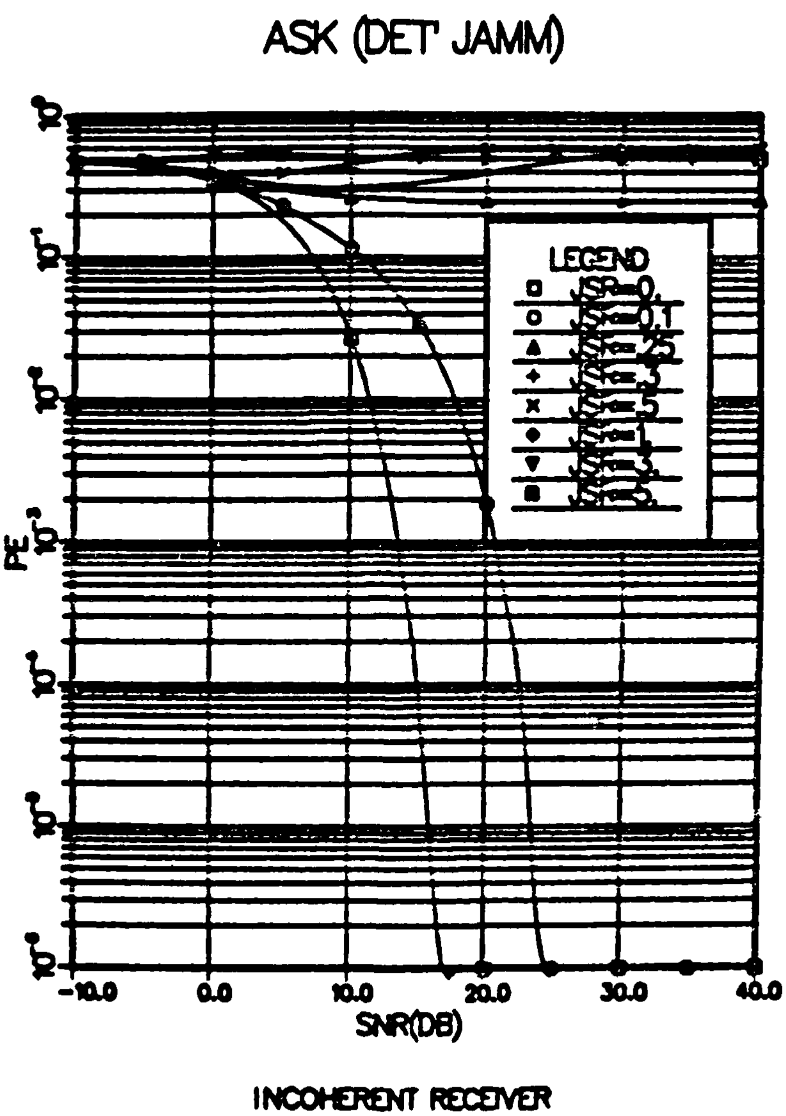

Figure 5.12 Performance of the ASK Incoherent Receiver with Deterministic Jamming

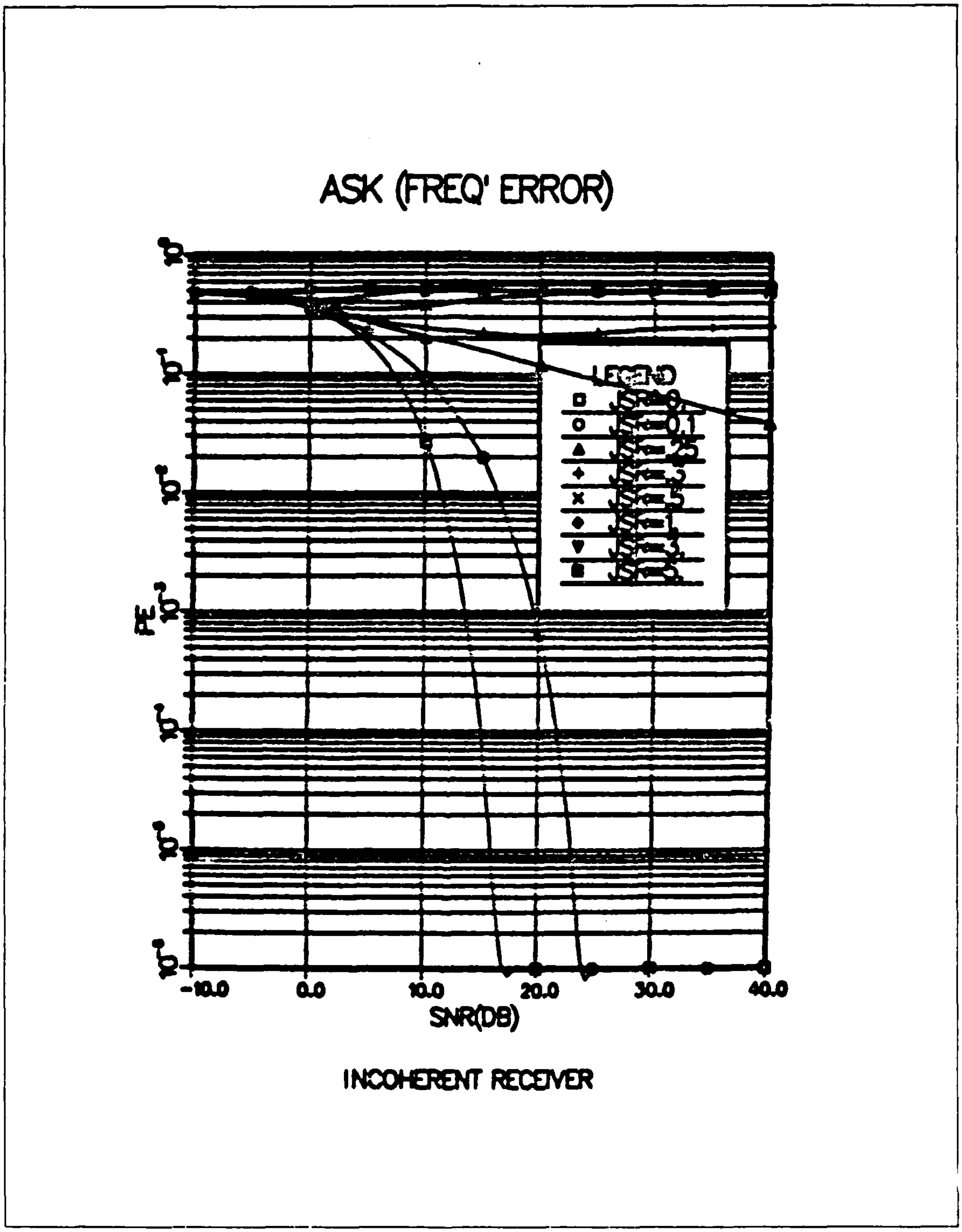

Figure 5.13 Performance of the ASK Incoherent Receiver with Frequency Error Jamming

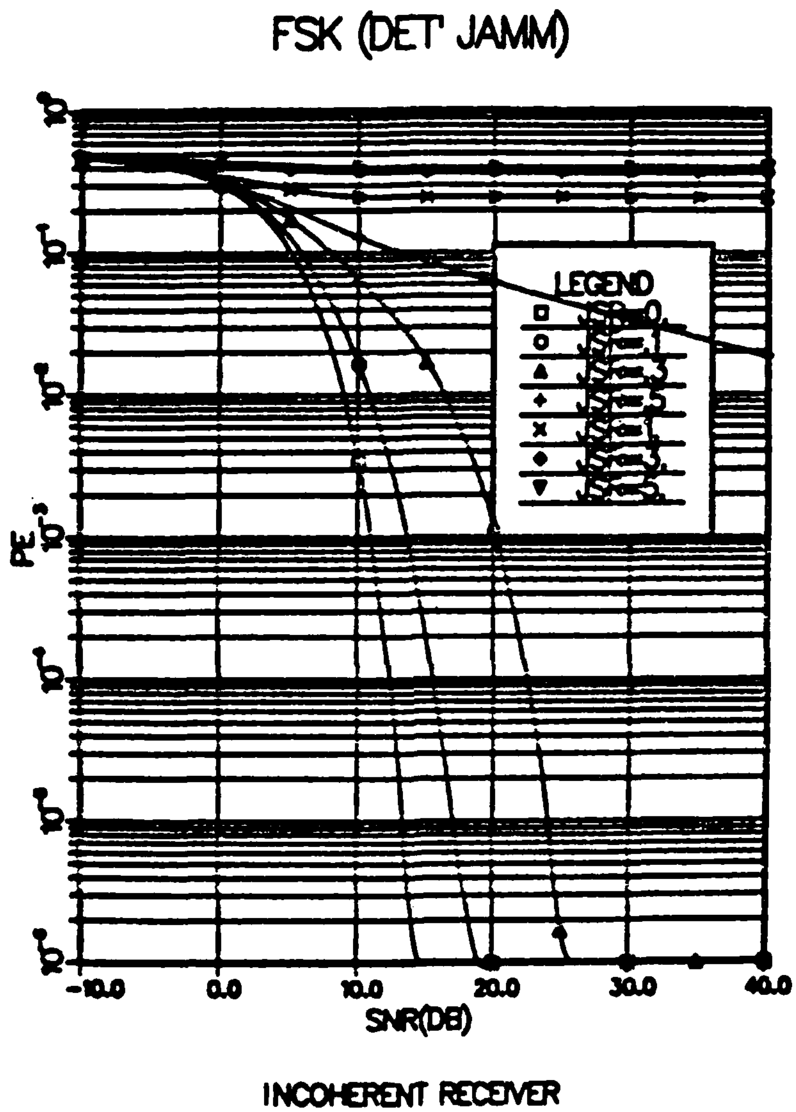

Figure 5.14 Performance of the FSK Incoherent Receiver with Deterministic Jamming

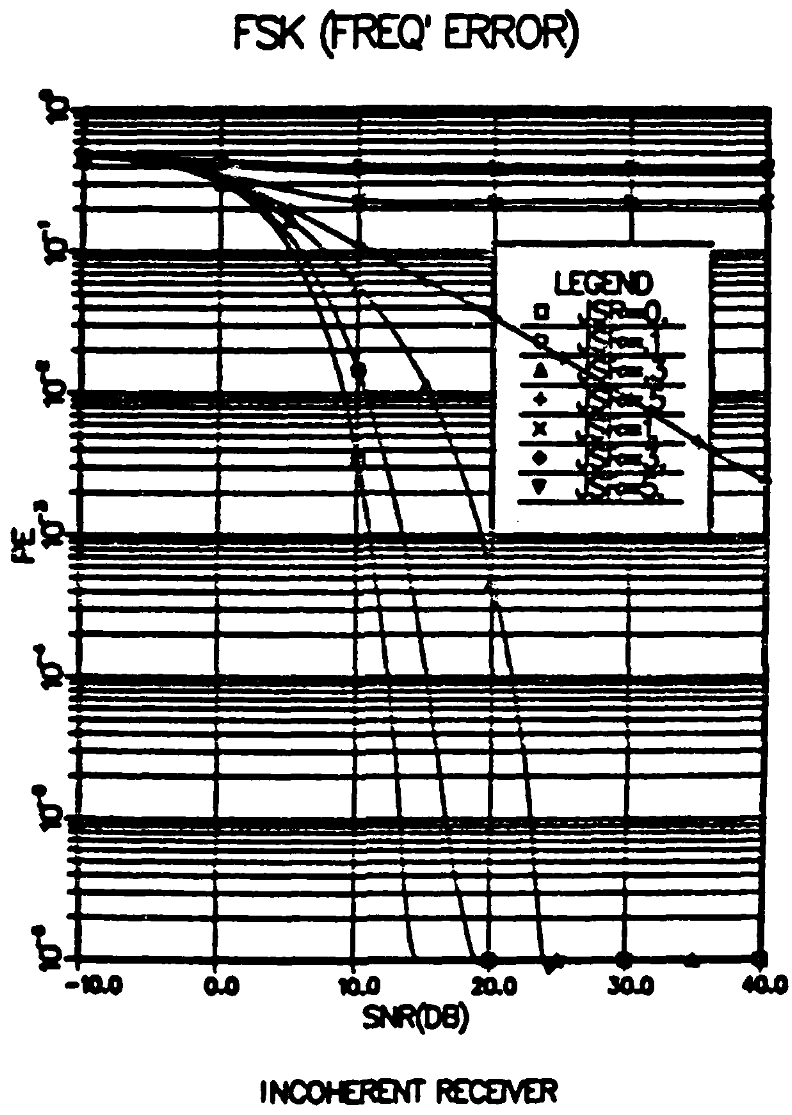

Figure 5.15 Performance of the FSK Incoherent Receiver with Frequency Error Jamming

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